



**Warning:** Subroutine EB08 performs functions which are adequately treated by routines in other standard subroutine libraries (for example, LAPACK). The use of this routine is not recommended, and it may be removed from future releases of this library.

## 1 SUMMARY

Finds **all the eigenvalues** and **right eigenvectors** of a **real  $n$  by  $n$  upper Hessenberg** matrix, i.e. finds solutions  $\lambda_i$  and  $\mathbf{z}_i$  of

$$\mathbf{H}\mathbf{z}_i = \lambda_i \mathbf{z}_i \quad i=1,2,\dots,n$$

where  $\mathbf{H}=\{h_{ij}\}_{n \times n}$  is a real matrix which has all zero elements  $h_{ij}=0$  for  $i>j+1$ . The eigenvectors are normalized to have unit length and there is an option which allows isolated eigenvalues to be specified.

A QR method is used, see J.H. Wilkinson and C. Reinsch, 'Handbook for Automatic Computation, Linear algebra', Springer-Verlag.

**ATTRIBUTES** — **Version:** 1.0.0. **Types:** EB08A; EB08AD. **Calls:** FD05; \_DOT. **Original date:** June 1973. **Origin:** HQR2 from reference given above, modified for Harwell by S.Marlow.

## 2 HOW TO USE THE PACKAGE

### 2.1 The argument list and calling sequence

*The single precision version:*

```
CALL EB08A(H, IH, Z, IZ, E, N, LFAIL)
```

*The double precision version:*

```
CALL EB08AD(H, IH, Z, IZ, E, N, LFAIL)
```

**H** is a two-dimensional REAL (DOUBLE PRECISION in the D version) array of dimensions at least  $n \times n$  (first dimension specified in **IH**), which must be set by the user to contain the elements of the matrix. The elements below the sub-diagonal need not be set to zero (since the matrix is upper Hessenberg) as they are assumed to be zero. The matrix including the part below the sub-diagonal is overwritten by the subroutine.

**IH** is an INTEGER variable which must be set by the user to the first dimension of the array **H**. **Restriction:**  $\text{IH} \geq n$ .

**Z** is a two-dimensional REAL (DOUBLE PRECISION in the D version) array of dimensions at least  $n \times n$  (first dimension specified in **IZ**) which will be set by the subroutine to contain the eigenvectors. If the  $i^{\text{th}}$  eigenvalue is real then the  $i^{\text{th}}$  column of **Z** will contain the corresponding eigenvector. If the  $i^{\text{th}}$  and  $(i+1)^{\text{th}}$  eigenvalues are a complex conjugate pair the columns  $i$  and  $i+1$  of **Z** give the real and imaginary parts of the eigenvector corresponding to the eigenvalue with positive imaginary part (the imaginary part of  $\mathbf{E}(i)$  will be zero if the  $i^{\text{th}}$  eigenvalue is real).

**IZ** is an INTEGER variable which must be set by the user to the first dimension of the array **Z**. **Restriction:**  $\text{IZ} \geq n$ .

**E** is a two-dimensional REAL (DOUBLE PRECISION in the D version) array of first dimension 2 which will be set by the subroutine to contain the eigenvalues.  $\mathbf{E}(1, \mathbf{I})$ ,  $\mathbf{E}(2, \mathbf{I})$  containing the real and imaginary parts of the  $\mathbf{I}^{\text{th}}$  eigenvalue.

**N** is an INTEGER variable which must be set by the user to  $n$  the order of the matrix.

LFAIL is an INTEGER variable and is an error return parameter and will be set by the subroutine to have one of the following values

- 0 successful return
- 1 value of N was non positive
- $k > 0$  the QR method failed to converge for the  $k^{\text{th}}$  eigenvalue

## 2.2 The Common area

The subroutine uses a common area which the user may reference by including a statement of the form

*The single precision version:*

```
COMMON/EB08B/ LP, ITSM, LL, LU, JH
```

*The double precision version:*

```
COMMON/EB08BD/ LP, ITSM, LL, LU, JH
```

LP is an INTEGER variable (default 6, line printer) that refers to the stream number on which diagnostic messages appear. To suppress these messages set LP to 0.

ITSM is an INTEGER variable (default 30) and is the limit on the number of iterations in finding an eigenvalue.

LL, LU are INTEGER variables (default 0,0) and indicate that the upper Hessenberg has isolated eigenvalues in positions 1 to LL-1 and LU+1 to N. A value of 0 for LL is regarded as the same as LL and LU having the values 1 and N respectively.

JH is an INTEGER variable (default 0) indicates that on entry to EB08A/AD the eigenvector array Z is initialised to the identity matrix. If it has a nonzero value then it is assumed that Z has already been set (i.e. the result of some transformation).

## 3 GENERAL INFORMATION

**Use of Common:** One Common area, EB08B/BD, is used (see §2.2).

**Workspace:** none.

**Other subroutines:** calls FD05, \_DOT.

**Input/Output:** error messages may be printed (see §2.2).

## 4 METHOD

The eigenvalues are found by applying the QR method. At the same time the eigenvectors are formed by accumulating the transformations. See Handbook for Automatic Computation, Linear Algebra, J.H. Wilkinson, C. Reinsch, Springer-Verlag.