

## 1 SUMMARY

To calculate  $\mathbf{A}^\dagger$  the **generalized inverse** of an  $m$  by  $n$  ( $m \leq n$ ) rectangular matrix  $\mathbf{A}$  in the special case that the **rank of  $\mathbf{A}$  is equal to  $m$** , i.e. such that  $\mathbf{A}\mathbf{A}^\dagger\mathbf{A} = \mathbf{A}$  which with full rank can be defined as  $\mathbf{A}^\dagger = \mathbf{A}^T(\mathbf{A}\mathbf{A}^T)^{-1}$ .

Householder type orthogonal transformations with row and column interchanges are used in a method described in M.J.D. Powell, AERE R.6072.

**ATTRIBUTES** — **Version:** 1.0.0. **Types:** MB11A; MB11AD. **Original date:** May 1969. **Origin:** M.J.D.Powell, Harwell.

## 2 HOW TO USE THE PACKAGE

### 2.1 The argument list and calling sequence

*The single precision version:*

```
CALL MB11A(M,N,A,IA,W)
```

*The double precision version:*

```
CALL MB11AD(M,N,A,IA,W)
```

**M** is an INTEGER variable set to  $m$  the number of rows in the matrix  $\mathbf{A}$ .

**N** is an INTEGER variable set to  $n$  the number of columns in the matrix  $\mathbf{A}$ .

**A** is a REAL (DOUBLE PRECISION in the D version) two dimensional array which must be set to contain the elements of the matrix  $\mathbf{A}$ . i.e.  $A(I,J) = a_{ij}$   $I=1,2,\dots,M$ ,  $J=1,2,\dots,N$ .

On exit the array  $\mathbf{A}$  will have been overwritten by its generalised inverse so that  $A(I,J)$  will be changed to the  $(I,J)$  th element of  $\mathbf{A}^{\dagger T}$ .

**IA** is an INTEGER variable set to the first dimension of the array  $\mathbf{A}$ . Note that we must have  $IA \geq M$ .

**W** is a REAL (DOUBLE PRECISION in the D version) workspace array of length at least  $2m+n$

## 3 GENERAL INFORMATION

**Use of Common:** none.

**Workspace:** all supplied by the user in the arrays  $\mathbf{W}$ .

**Other subroutines:** None

**Input/Output:** none.

## 4 METHOD

First  $\mathbf{A}$  is transformed to a lower triangular form, by a sequence of  $m$  elementary Householder transformations, taking account of row and column interchanges. This lower triangular matrix is inverted, and then it is replaced by another matrix that contains the same information in a more convenient form. Because of this replacement, we can now re-apply the elementary transformations to the inverted matrix, to obtain the required generalised inverse, without requiring extra storage space. The method is given in M.J.D.Powell, 'A Fortran subroutine to invert a rectangular matrix of full rank', AERE Report R-6072.