1 SUMMARY

This subroutine generates pseudo-random numbers from the gamma distribution, \( G(\alpha, \beta) \), with shape parameter \( \alpha \) and scale parameter \( \beta \). The distribution has the probability density function (p.d.f.)

\[
f(x) = \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta} \quad \alpha \geq 1 \quad \beta > 0 \quad 0 \leq x < \infty.
\]

The mean \( \mu \) and variance \( \sigma^2 \) of the distribution are given by

\[
\mu = \alpha \beta,
\]

and

\[
\sigma^2 = \alpha \beta^2.
\]

The special case \( \alpha=1 \) corresponds to the exponential distribution with p.d.f.

\[
g(x) = \frac{1}{\beta} e^{-x/\beta} \quad \beta > 0 \quad 0 \leq x < \infty
\]

so that the subroutine can also be used for generating exponentially distributed random numbers.

For \( \alpha > 1 \), the subroutine uses the ratio-of-uniforms method for generating random numbers with continuous non-uniform distributions. For the case \( \alpha = 1 \), the standard log-uniform method is used. Full details are given in Robertson, I. and Walls, L.A., Harwell report CSS.89, (1980).


2 HOW TO USE THE PACKAGE

2.1 Calling sequence and argument list

\[
\text{CALL FA06A(ALPHA,BETA,Z)}
\]

\text{ALPHA} is a REAL variable which must be set by the user to the shape parameter \( \alpha \) of the gamma distribution. This argument is not altered by the subroutine. Restriction: \( \alpha \geq 1 \), see also §2.2.

\text{BETA} is a REAL variable which must be set by the user to the scale parameter \( \beta \) of the gamma distribution. The sign of the variable \( \text{BETA} \) is not significant, since the subroutine works only with its absolute value. This argument is not altered by the subroutine.

\text{Z} is a REAL variable. Following a successful exit from the subroutine, \( Z \) contains a pseudo-random number from the gamma distribution with parameters \( \alpha \) and \( \beta \). However, if an illegal \( \alpha \)-value is provided (\( \alpha < 1 \)), \( Z \) will be set to \(-1.0\) on exit, see also §2.2.
2.2 Common area and error diagnostics

The shape parameter, $\alpha$, supplied to FA06A must satisfy $\alpha \geq 1$. If an illegal value is passed the subroutine returns a value of $-1.0$ for the pseudo-random number, $z$, and prints out a diagnostic message. The user may control the production and destination of this message by adjusting a variable, IPRINT, held in the labelled common block

```plaintext
COMMON/FA06B/ IPRINT
```

IPRINT is an INTEGER variable (default value 6 for line printer) which may be set by the user to be

- $> 0$ to get diagnostic messages printed on Fortran stream unit number IPRINT.
- $\leq 0$ to suppress the printing of diagnostic messages.

3 GENERAL INFORMATION

Use of common: uses FA06B, see §2.2.

Workspace: none.

Other routines: the library routine FA04AS is used for generating random numbers uniformly distributed on the interval (0,1).

Input/Output: diagnostic messages are printed, see §2.2.

Restriction: $\alpha \geq 1$.

4 METHOD

For $\alpha > 1$ the subroutine uses the ratio of uniforms method for generating random numbers with a continuous non-uniform distribution. In this method, an acceptance-rejection technique is used to generate a point uniformly over the plane region defined by the inequality

$$y \leq x(\alpha - 1) \ln y - (\alpha + 1) \ln x.$$ 

The ratio of the coordinate values of this point yields a random variable, $s$, from the gamma distribution $G(\alpha, 1)$. A variable from $G(\alpha, \beta)$ is then obtained by the transformation

$$z = \beta s.$$ 

For the case, $\alpha = 1$, the standard log-uniform method is used. The theory underlying the method is described in the references given below.

References
