1 SUMMARY

This subroutine **generates pseudo-random numbers from the gamma distribution,** $G(\alpha,\beta)$, with shape parameter α and scale parameter β. The distribution has the probability density function (p.d.f.)

$$f(x)=\frac{1}{\beta\Gamma(\alpha)}\left(\frac{x}{\beta}\right)^{\alpha-1}e^{-\frac{x}{\beta}}\qquad \alpha\geq 1\qquad \beta>0\qquad 0\leq x<\infty.$$

The mean μ and variance σ^2 of the distribution are given by

$\mu=\alpha\beta,$

and

$\sigma^2=\alpha\beta^2.$

The special case α=1 corresponds to the **exponential distribution** with p.d.f.

$$g(x)=\frac{1}{\beta}e^{-\frac{x}{\beta}}\qquad \beta>0\qquad 0\leq x<\infty$$

so that the subroutine can also be used for generating exponentially distributed random numbers.

For $\alpha>1$, the subroutine uses the ratio-of-uniforms method for generating random numbers with continuous non-uniform distributions. For the case $\alpha=1$, the standard log-uniform method is used. Full details are given in Robertson, I. and Walls, L.A., Harwell report CSS.89, (1980).

ATTRIBUTES — **Version:** 1.0.0. **Types:** `FA06A`; there is no double precision version. **Calls:** `FA04` and `FD05`. **Original date:** September 1980. **Origin:** I.Robertson and L.A.Walls*, Harwell.

2 HOW TO USE THE PACKAGE

2.1 Calling sequence and argument list

```
CALL FA06A(ALPHA,BETA,Z)
```

`ALPHA` is a `REAL` variable which must be set by the user to the shape parameter α of the gamma distribution. This argument is not altered by the subroutine. **Restriction:** $\alpha\geq 1$, see also §2.2.

`BETA` is a `REAL` variable which must be set by the user to the scale parameter β of the gamma distribution. The sign of the variable `BETA` is not significant, since the subroutine works only with its absolute value. This argument is not altered by the subroutine.

`Z` is a `REAL` variable. Following a successful exit from the subroutine, `Z` contains a pseudo-random number from the gamma distribution with parameters α and β. However, if an illegal α–value is provided ($\alpha<1$), `Z` will be set to –1.0 on exit, see also §2.2.

2.2 Common area and error diagnostics

The shape parameter, α, supplied to `FA06A` must satisfy $\alpha \geq 1$. If an illegal value is passed the subroutine returns a value of –1.0 for the pseudo-random number, `Z` , and prints out a diagnostic message. The user may control the production and destination of this message by adjusting a variable, `IPRINT`, held in the labelled common block

```
COMMON/FA06B/ IPRINT
```

`IPRINT` is an `INTEGER` variable (default value 6 for line printer) which may be set by the user to be

> 0 to get diagnostic messages printed on Fortran stream unit number `IPRINT`.

≤ 0 to suppress the printing of diagnostic messages.

3 GENERAL INFORMATION

Use of common: uses `FA06B`, see §2.2.

Workspace: none.

Other routines: the library routine `FA04AS` is used for generating random numbers uniformly distributed on the interval (0,1).

Input/Output: diagnostic messages are printed, see §2.2.

Restriction: $\alpha \geq 1$.

4 METHOD

For $\alpha > 1$ the subroutine uses the ratio of uniforms method for generating random numbers with a continuous non-uniform distribution. In this method, an acceptance-rejection technique is used to generate a point uniformly over the plane region defined by the inequality

$$y \leq x\,(\alpha - 1)\ln y - (\alpha + 1)\ln x\,.$$

The ratio of the coordinate values of this point yields a random variable, s, from the gamma distribution $G(\alpha, 1)$. A variable from $G(\alpha, \beta)$ is then obtained by the transformation

$$z = \beta s.$$

For the case, $\alpha = 1$, the standard log-uniform method is used. The theory underlying the method is described in the references given below.

References

Kinderman, A.J. and Monahan, J.F., ‘Computer Generation of Random Variables using the Ratio of Uniform Deviates’, A.C.M. TOMS, Vol. 3, No. 3, (1977), pp 257-260.

Robertson, I. and Walls, L.A. ‘Random Number Generators for the Normal and Gamma Distributions using the Ratio of Uniforms Method’, Harwell report CSS.89, (1980).