1 SUMMARY

To compute values of the **Gamma function**

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t}\, dt$$

In the range $2 \le x \le 3$ an approximation of the form

$$\sum_{n=0}^{15} a_n (x-2)^n$$

is used; for $x > 10$ Stirling's approximation is used including up to 10 terms of the asymptotic expansion.

For other values except $x = 0$ or a negative integer the relationship $\Gamma(x+1) = x\,\Gamma(x)$ is used to relate the required value with the range $2 \le x \le 3$.

ATTRIBUTES — **Version:** 1.0.0. **Types:** `FC03A`; `FC03AD`. **Original date:** March 1963. **Origin:** S.Marlow, Harwell.

2 HOW TO USE THE PACKAGE

The single precision version

```
CALL FC03A(G,X)
```

The double precision version

```
CALL FC03AD(G,X)
```

`G` is a `REAL` (`DOUBLE PRECISION` in the D version) variable which will be set by the subroutine to the computed value of the function $\Gamma(x)$.

`X` is a `REAL` (`DOUBLE PRECISION` in the D version) variable which must be set by the user to the value of x for which the function is to be calculated. **Restrictions:** x must not be a negative integer, x must not be zero, and it must not be so large as to cause overflow.

When x is negative the function $\Gamma(x)$ is defined formally by the recurrence relation given in §4 (iii).

3 GENERAL INFORMATION

Use of common: none.

Workspace: none.

Other routines called directly: none.

Input/output: none.

Restrictions:

$x \ne$ negative integer,

$x \ne 0$.

Accuracies:

6 figures using 4-byte arithmetic

14 figures using 8-byte arithmetic

4 METHOD

The following approximations are used

(i) $2 \le x \le 3$, a Chebyshev polynomial approximation of degree fifteen is used.

(ii) $x > 10$, an asymptotic expansion of the form

$$\ln \Gamma(x) = \ln \sqrt{2\pi} + (x - \tfrac{1}{2}) \ln x - x + \sum_{r=1}^{10} \frac{b_r}{x^{2r-1}}$$

is used.

(iii) For further values of x the recurrence relation

$$\Gamma(x+1) = x\,\Gamma(x)$$

is used to relate the required value to one in the range $2 \le x \le 3$.