1 SUMMARY

Computes the real and imaginary parts of the Plasma Dispersion Function

\[ Z(z) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-t^2}}{t-z} \, dt \]

where \( z = x + iy \), for the case \( y > 0 \), and the analytic continuation of this for \( y < 0 \) as defined by Fried and Conte, ‘The Plasma Dispersion Function’, Academic Press, 1961. The derivative \( Z'(z) = -2x(1+zZ(z)) \) is also computed.

If \( y \geq 2.75 \) or if \( y \geq 2 \) and \( x \geq 4 \) an asymptotic continued fraction due to Fried and Conte is used, otherwise if \( x \geq 6.25 \) a rational approximation from Abramowitz and Stegun is used, otherwise a Taylor series is used.


2 HOW TO USE THE PACKAGE

The single precision version

CALL FC12A(X,Y,ZR,ZI,ZPR,ZPI)

The double precision version

CALL FC12AD(X,Y,ZR,ZI,ZPR,ZPI)

X is a REAL (DOUBLE PRECISION in the D version) variable which must be set by the user to the value of \( x \) the real part of the argument \( z = x + iy \).

Y is a REAL (DOUBLE PRECISION in the D version) variable which must be set by the user to the value of \( y \) the imaginary part of the argument \( z = x + iy \).

ZR is a REAL (DOUBLE PRECISION in the D version) variable which is set by the subroutine to the real part of the computed value of the function \( Z(z) \).

ZI is a REAL (DOUBLE PRECISION in the D version) variable which is set by the subroutine to the imaginary part of the computed value of the function \( Z(z) \).

ZPR is a REAL (DOUBLE PRECISION in the D version) variable which is set by the subroutine to the real part of the computed value of the first derivative \( Z'(z) \).

ZPI is a REAL (DOUBLE PRECISION in the D version) variable which is set by the subroutine to the imaginary part of the computed value of the first derivative \( Z'(z) \).

3 GENERAL INFORMATION

Use of common: none.

Workspace: none.

Other subroutines: none.

Input/Output: none.

Restrictions: none.
Accuracies: approx. $10^{-6}$ absolute.

4 METHOD

For $x, y \geq 0$ one of the following three methods is used

(i) If $y \geq 2.75$, or if $y \geq 2$ and $x \geq 4$: the subroutine uses the asymptotic continued fraction given by B.D. Fried and S.D. Conte, “The Plasma Dispersion Function”, Academic Press, 1961. [Note that some of the signs in the tables for $y < 0$ are wrong in the report from which this book was derived.]

(ii) Otherwise if $x \geq 6.25$: the subroutine uses the rational function

$$Z(z) = \frac{-z}{z^2 - 0.2752551 + \frac{0.09175171}{z^2 - 2.724745}}$$


(iii) Otherwise: the subroutine uses a Taylor series expansion

$$Z(z) = \sum_{i=0}^{n} T(z)^{(i)}$$

about the nearest point $z_0$ on the mesh $x_0 = 0.0(0.5)6.0$, $y_0 = 0.0(0.2)2.5$ using the recurrences

$$T^{(0)} = Z(z_0),$$

$$T^{(1)} = (z - z_0) Z(z_0)$$

$$T^{(n+1)} = \left\{ \frac{z_0 T^{(n+1)} + (z - z_0) T^{(n)}}{i n + 1} \right\}$$

For $y < 0$ with $x > 0$ the relationship

$$Z(x-iy) = 2\sqrt{\pi e^{-y^2}} \left\{ -\sin(2x|y|) + ... + i\cos(2|y|) \right\} + \text{conj}\{Z(x+iy)\}$$

is used, and for $x < 0$ the relationship

$$Z(-x+iy) = -\text{conj}\{Z(x+iy)\}$$

is used. The boundaries are chosen so that the results agree with the tables in Fried and Conte on a mesh $x = \pm 0.0(0.2)7.0$, $y = 0.0(0.2)4.0$ to an absolute accuracy in both real $\{Z(z)\}$ and imag $\{Z(z)\}$ of $10^{-6}$ or better. There is good reason to think that a similar accuracy holds for larger $x$ and $y$. For $y < 0$ the error is dominated by the first term of the formula (4.1) and is better than about $2 \times 10^{-6} \times 2\sqrt{\pi} e^{x^2} e^{-y^2}$.