1 SUMMARY

To compute values of the **natural logarithm of the Gamma function,** i.e. $\ln|\Gamma(x)|$ where

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t}\, dt$$

The calculation is performed to high precision over a wide range of argument values but excluding those in the neighbourhood of zero and the negative integers.

ATTRIBUTES — **Version:** 1.0.0. **Remark:** `FC14` is to be preferred to `FC03` when calculating values of $\Gamma(x)$. **Types:** `FC14A`; `FC14AD`. **Original date:** November 1982. **Origin:** A.R.Curtis, Harwell.

2 HOW TO USE THE PACKAGE

2.1 Computing Gamma Function Values

The logarithm of $\Gamma(x)$ is the natural function to use, as $\Gamma(z)$ itself can overflow or underflow for $|x|>56$; the absolute value must be taken since the sign of $\Gamma(x)$ is $(-1)^n$ for $-n<x<1-n$. If $\Gamma(z)$ is required (and will not overflow), it can be obtained by using the Fortran exponential function subroutine `DEXP` after return from `FC14`, then truncating the result to single precision if desired.

2.2 The Argument List and Calling Sequence

The single precision version

```
CALL FC14A(X,Y)
```

The double precision version

```
CALL FC14AD(X,Y)
```

`X` is a `REAL` (`DOUBLE PRECISION` in the D version) variable which must be set by the user to the value of x for which the function is to be calculated. This argument is not altered by the subroutine. **Restrictions:** x must not be a negative integer, zero, or close enough to any of these values so as to cause underflow.

When x is negative the function $\Gamma(x)$ is defined formally by the recurrence relation given in (iii) in §4.

`Y` is a `REAL` (`DOUBLE PRECISION` in the D version) variable which will be set by the subroutine to the computed value of the natural logarithm of $\Gamma(x)$.

3 GENERAL INFORMATION

Use of common: none.

Workspace: none.

Other routines called directly: none.

Input/output: none.

Restrictions:

$x \neq$ negative integer,
$x \neq 0$,

or close enough to any of these values to cause underflow.

Accuracies:

6 figures using 4-byte arithmetic.

$< \max\{1, \ln|\Gamma(x)|\} \times 10^{-15}$ using 8-byte arithmetic.

4 METHOD

The following approximations are used

(i) $2 \le x \le 3$, a Chebyshev series is used and the logarithm taken.

(ii) $x \ge 6$, an asymptotic expansion of the form

$$\ln \Gamma(x) = \ln \sqrt{2\pi} + (x - \tfrac{1}{2}) \ln x - x + \sum_{r=1}^{10} \frac{b_r}{x^{2r-1}}$$

is used.

(iii) if $-15 \le x < 6$ and x is not in the range the recurrence relation

$$\Gamma(x+1) = x\,\Gamma(x)$$

is used to relate the required value to one in the range

(iv) if $x < -15$, the relation

$$\Gamma(x)\Gamma(1-x) = \pi \operatorname{cosec} \pi x$$

is used, computing $-\ln \Gamma(1-x)$ from the asymptotic expansion and calling `DLOG` and `DSIN` to compute a correction term.

The error is less than $\max(1, |\ln \Gamma(x)|) \times 10^{-15}$ for all cases tested covering (with varying density) the range $-10^{15} < x < 10^{15}$.