1 SUMMARY

To compute \( \theta = \frac{w(r,h)}{2\pi} \), where \( w(r,h) \) is the solid angle subtended by a disc of unit radius from a general point \((r,h)\) in the plane perpendicular to the plane of the disc.

On the plane of the disc \((h=0)\): \( \theta = 1 \) for \( r < 1 \), \( \theta = 0 \) for \( r > 1 \), and \( \theta = \pm \frac{1}{2} \) at the undefined point \( r = 1 \).

The integral expression for the solid angle is represented in terms of complete and incomplete elliptic integrals of the first kind, see M. Ruffle, AERE R.5419.


2 HOW TO USE THE PACKAGE

2.1 The argument list

The single precision version

CALL GA04A(R,H,THETA)

The double precision version

CALL GA04AD(R,H,THETA)

\( R \) is a REAL (DOUBLE PRECISION in the D version) variable which must be set by the user to the \( r \) co-ordinate of the point. Restriction: \( r \geq 0 \).

\( H \) is a REAL (DOUBLE PRECISION in the D version) variable which must be set by the user to \( h \) the height of the point above the plane of the disc. Restriction: \( h \geq 0 \).

\( \text{THETA} \) is a REAL (DOUBLE PRECISION in the D version) variable which is set by the subroutine to \( \theta = \frac{w(r,h)}{2\pi} \), where \( w(r,h) \) is the solid angle.

3 GENERAL INFORMATION

Use of common: None.
Workspace: None.
Other routines called directly: FB01A/AD and FB02A/AD.
Input/output: None.
Restrictions: \( r \geq 0, \ h \geq 0 \).

4 METHOD

The required expressions for the solid angle \( w(r,h) \) are:

- for \( r \geq 1 \)
  \[
  w(r,h) = \frac{1}{2\pi} I(a,b)
  \]
- for \( r < 1 \)
  \[
  w(r,h) = 1 - \frac{h}{\sqrt{h^2+(1-r)^2}} + \frac{1}{2\pi} I(a,b)
  \]

where

\[
I(a,b) = 2 \int_{a}^{b} \sin \phi \cos^{-1} \left( \frac{r^2 + h^2 \tan^2 \phi - 1}{2rh \tan \phi} \right) d\phi
\]

and where

\[
a = \tan^{-1} \left( \frac{|r-1|}{h} \right)
\]

and

\[
b = \tan^{-1} \left( \frac{r+1}{h} \right)
\]

These expressions may be written in terms of complete and incomplete Elliptic integrals of the first and second kinds, see Ruffle. M., “The computation of certain solid angles and some useful associated functions”, Harwell Report R-5419.