1 SUMMARY

Calculates **the minimax solution** of a system of m **linear algebraic equations** in n unknowns, $m \geq n$, where the maximum element of the solution is **subject to a simple bound.** Given equations

$$\sum_{j=1}^{n} a_{ij} x_j + b_i = 0 \qquad i=1,2,...,m \quad m \geq n$$

find the solution x_j j=1,2,...,n such that

$$\max_i \left\{ \left| \sum_{j=1}^{n} a_{ij} x_j + b_i \right| \right\}$$

is minimized subject to the bounds $|x_j| \leq g$ j=1,2,...,n.

A variation of the 'exchange algorithm' is used that incorporates a technique for reducing the number of iterations, and which will also provide a defined solution even when the matrix $\mathbf{A} = \{a_{ij}\}_{n \times n}$ is rank deficient.

Described in K.Madsen and M.J.D.Powell, Harwell report R.7954 (1975).

ATTRIBUTES — **Version:** 1.0.0. **Types:** MA19A; MA19AD. **Original date:** March 1974. **Origin:** K.Madsen, Copenhagen.

2 HOW TO USE THE PACKAGE

2.1 The argument lists

The single precision version

```
CALL MA19A(N,M,A,IA,B,G,EPS,X,RES,IREF)
```

The double precision version

```
CALL MA19AD(N,M,A,IA,B,G,EPS,X,RES,IREF)
```

N is an INTEGER variable and must be set by the user to n, the number of unknowns. **Restriction:** $n > 0$.

M is an INTEGER variable and must be set to m, the number of linear expressions under consideration. **Restriction:** $m > 0$.

A is a REAL (DOUBLE PRECISION in the D version) two-dimensional array which the user must set to the coefficients a_{ij}, i.e. A(i,j) $= a_{ij}$, i=1,2,...,m, j=1,2,...,n. It is not changed by the subroutine.

IA is an INTEGER variable and must be set to the first dimension of the array A.

B is a REAL (DOUBLE PRECISION in the D version) array which the user must set to the constants b_i, i.e. B(i) $= b_i$, i=1,2,...,m. It is not changed by the subroutine.

G is a REAL (DOUBLE PRECISION in the D version) variable which must be set to the value of the bound g on the unknowns $\mathbf{x}_j$. **Restriction:** $g \geq 0$.

EPS is a REAL (DOUBLE PRECISION in the D version) variable which controls the accuracy of the solution, i.e. the solution will satisfy equation (3) in section 4 with ε= EPS. It has to be set to a non-negative value by the user, and normally EPS=0 will be a good value.

X is a REAL (DOUBLE PRECISION in the D version) array in which the subroutine will return the solution. The array should be of length at least n.

`RES` is a `REAL` (`DOUBLE PRECISION` in the D version) array which is used for working space. Its length must be at least $(n+1)(n+5)+m$. On exit it will contain the values

$$\texttt{RES(i)} = \sum_{j=1}^{n} a_{ij} x_j + b_i, \qquad i=1,2,...,m.$$

Further, `RES(`m+1) will contain the value of h in equation (1) in section 4, and `RES(`m+2) will contain an upper bound for the expressions on the left-hand side of equation (3) in section 4.

`IREF` is an `INTEGER` array which is used for working space. Its length must be at least $4(n+1)+m$. On exit |`IREF(i)`|, $i=1,2,...,(n+1)$, will contain information about which equations belong to the sets I_D and I_B defined in section 4.

|`IREF(i)`|$>m$ will mean that the index |`IREF(i)`|$-m$ is in the set I_B. Otherwise |`IREF(i)`| belongs to the set I_D. The sign of `IREF(i)` is the sign of s_i in equations (1) and (2) in section 4. Further `IREF(n+2)` will give the number of elements in I_B, and `IREF(n+3)` will give the number of iterations used by the method.

3 GENERAL INFORMATION

Use of common: None.

Workspace: Provided by the user, see arguments `RES` and `IREF`.

Other routines called directly: None.

Input/output: None.

Restrictions: $n \geq 1$, $m \geq 1$, $g \geq 0$, $\varepsilon \geq 0$.

4 METHOD

The method will be described in a forthcoming Harwell report. The solution to the problem will be the solution to $(n+1)$ linear equations:

$$\sum_{j=1}^{n} a_{ij} x_j + b_i = s_i h, \qquad \text{for } i \in I_D \tag{1}$$

$$x_i = s_i g_1 \qquad \text{for } i \in I_B \tag{1}$$

where $h \geq 0$, and $g_1 = g$ unless $h = 0$ when $g_1 \leq g$. Further $s_i = \pm 1$, and I_B contains n_b indices, I_D contains $(n+1-n_{b)}$ indices, $0 \leq n_b \leq n$. At the solution the inequalities

$$\left| \sum_{j=1}^{n} a_{ij} x_j + b_i \right| \leq h + \varepsilon \tag{3}$$

and

$$|x_k| \leq g$$

will be satisfied for all relevant values of i and k.

Note that if the bounds instead of being of the form $|x_j| \leq g$, $j=1,2,...,n$ are

$$|x_j| \leq g_j, \qquad j=1,2,...,n$$

the problem is easily re-written in the first form by scaling the coefficients a_{ij}.