Warning: Subroutine ME08 performs functions which are adequately treated by routines in other standard subroutine libraries (for example, LAPACK). The use of this routine is not recommended, and it may be removed from future releases of this library.

1 SUMMARY

To transform by Householders method, a **complex Hermitian matrix** ($\mathbf{A}=\{a_{ij}\}$, $a_{ij}=\bar{a}_{ij}$), to a tri-diagonal matrix which has the same eigenvalues, i.e. given an $m \times m$ Hermitian matrix $\mathbf{A}=\mathbf{A}^{\mathrm{H}}$ with eigenvalues λ defined by $\mathbf{Ax}=\lambda\mathbf{x}$, transforms A to the matrix

$$\mathbf{T} = \begin{bmatrix} \alpha_1 & \beta_2 & & & & \\ \bar{\beta}_2 & \alpha_2 & \beta_3 & & & \\ & \bar{\beta}_3 & \alpha_3 & \beta_4 & & \\ & & * & * & * & \\ & & & \bar{\beta}_{m-1} & \alpha_{m-1} & \beta_m \\ & & & & \bar{\beta}_m & \alpha_m \end{bmatrix}$$

where $\bar{\beta}_j$ is the complex conjugate of β_j and $\mathbf{Tu}=\lambda\mathbf{u}$ defines the same eigenvalues.

ATTRIBUTES — Version: 1.0.0. **Types:** ME08A; ME08AD. **Original date:** February 1966. **Language:** ME08AD uses COMPLEX*16 facility. **Origin:** S.Marlow, Harwell.

2 HOW TO USE THE PACKAGE

2.1 The argument list

The single precision version

```
CALL ME08A(A,ALPHA,BETA,M,IA,WORK)
```

The double precision version

```
CALL ME08AD(A,ALPHA,BETA,M,IA,WORK)
```

A is a COMPLEX (COMPLEX*16 in the D version) (see §2.2) two dimensional array of first dimension IA in which the user must set the elements of the real symmetric matrix $\mathbf{A}=\{a_{ij}\}$.

Only the lower triangle elements A(I,J), I ≥ J need be set and these are not overwritten by the subroutine. The imaginary parts of the diagonal elements of the array A are assumed by the subroutine to have been set to zero. The elements of the upper triangle are used as work space and will contain information on the rotations required necessary to recover the eigenvectors of **A** from those of the tri-diagonal matrix. If the k th rotation was $(I-\theta\bar{\mathbf{u}}_k\mathbf{u}_k^{\mathrm{T}})$ the non-zero elements of $\mathbf{u}_k$ are returned in A(K,I), I=K+1,.,N and the scaling is given by θ= 1/(CONJG(A(K,K+1))*BETA(K+1)).

ALPHA is a COMPLEX (COMPLEX*16 in the D version) (see §2.2) array of length at least m and is set by the subroutine to the diagonal elements α_i, i=1,2,...,m of the tri-diagonal matrix **T**. The imaginary parts will be set to zero.

BETA is a COMPLEX (COMPLEX*16 in the D version) (see §2.2) array of length at least m which is set by the subroutine to the super-diagonal elements β_i, i=2,3,..,m of the tri-diagonal matrix **T**. The sub-diagonal elements of the matrix should be assumed to be the complex conjugate of those given in BETA.

`M` is an `INTEGER` variable which must be set by the user to *m* the order of the matrix.

`IA` is an `INTEGER` variable which must be set by the user to the first dimension of the array `A`.

`WORK` is a `COMPLEX` (`COMPLEX*16` in the D version) (see §2.2) array of length at least *m* which is used by the subroutine as workspace.

2.2 The COMPLEX argument types

To conform to the Fortran 77 standard the array arguments listed as `COMPLEX` (`COMPLEX*16` in the D version) in §2.1 should strictly be `REAL` (`DOUBLE PRECISION` in the D version) arrays with an extra first dimension of 2, e.g.

```
      REAL A(2,10,10),ALPHA(2,10),BETA(2,10),WORK(2,10)
       -   -   -
      CALL ME08A(A,ALPHA,BETA,5,10,WORK)
```

Most implementations of Fortran allow such arguments to be passed in the more convenient form of `COMPLEX` (`COMPLEX*16` in the D version).

3 GENERAL INFORMATION

Use of common: None.

Workspace: See argument `WORK`.

Other routines called directly: None.

Input/output: None.

4 METHOD

Complex Householder transformations of the form ($I - \theta \bar{\mathbf{u}} \mathbf{u}^T$) are applied to the matrix and are calculated to reduce in $n-2$ stages the matrix to tri-diagonal form while at the same time preserving the eigenvalues, see Numerische Mathematik, Dec. 1962.