

## 1 SUMMARY

To find **all the real roots of a polynomial with real coefficients** that fall within a given interval  $x_1 \leq x \leq x_2$ , i.e. calculate the zeros of

$$a_0 + a_1x + a_2x^2 + \dots + a_nx^n = 0 \quad x_1 \leq x \leq x_2 \quad n \leq 49$$

The subroutine will also return the number of real roots above, below and within the interval and there is an option to request all the real roots.

Sturm's sequence polynomials are generated to bracket the roots then a combination of Newton-Raphson and bisection is used to refine each root. The subroutines provide an estimate of induced error growth by a method of feeding a perturbation through the recurrence relation that generated the Sturm's sequence.

**ATTRIBUTES** — **Version:** 1.0.0. **Types:** PA02B; PA02BD. **Original date:** September 1965. **Origin:** M.J.Hopper, Harwell.

## 1 SUMMARY

### 2.1 The argument list and calling sequence

*The single precision version*

```
CALL PA02B(A, X1, X2, NB, NR, NA, N, ROOT, E)
```

*The double precision version*

```
CALL PA02BD(A, X1, X2, NB, NR, NA, N, ROOT, E)
```

- A is a REAL (DOUBLE PRECISION in the D version) array which must be set by the user to the coefficients of the polynomial, i.e. set  $A(j+1) = a_j, j=0, 1, \dots, n$ . **Restriction:**  $a_n \neq 0$ .
- X1 is a REAL (DOUBLE PRECISION in the D version) variable which must be set by the user to the lower limit  $x_1$  of the search interval. See the description of X2 for more details. **Restriction:**  $x_1 \leq x_2$ .
- X2 is a REAL (DOUBLE PRECISION in the D version) variable which must be set by the user to the upper limit  $x_2$  of the search interval. The subroutine finds all the real roots in the interval  $x_1 \leq x \leq x_2$ . If X1 is set equal to X2 the subroutine will find all the real roots. **Restriction:**  $x_1 \leq x_2$ .
- NB is an INTEGER variable which the subroutine will set to the number of real roots below the limit  $x_1$ . If X1=X2 the value returned should be ignored.
- NR is an INTEGER variable which the subroutine will set to the number of real roots found in the interval  $x_1 \leq x \leq x_2$ . If X1=X2 the value of NR returned will be the total number of real roots.
- NA is an INTEGER variable which the subroutine will set to the number of real roots above the limit  $x_2$ . If X1=X2 the value returned should be ignored.
- N is an INTEGER variable which must be set by the user to  $n$  the degree of the polynomial. **Restriction:**  $0 \leq n \leq 49$ .
- ROOT is a REAL (DOUBLE PRECISION in the D version) array which will be returned containing the roots found in the interval  $x_1 \leq x \leq x_2$ . They will be ordered so that  $ROOT(1) \leq ROOT(2) \leq \dots \leq ROOT(NR)$ .
- E is a REAL (DOUBLE PRECISION in the D version) array which will be returned containing estimates of the errors in the roots. These are calculated by tracing the growth of a perturbation induced into the recurrence relation used for evaluating the polynomial and its derivative.

### 3 GENERAL INFORMATION

**Use of common:** None.

**Workspace:** Internal work arrays limit the degree  $n \leq 49$ .

**Other routines called directly:** None.

**Input/output:** None.

**Restrictions:**  $a_n \neq 0$ ,  $0 \leq n \leq 49$ ,  $x_1 \leq x_2$ .

### 4 METHOD

The subroutine is divided into three sections

(i) it forms a Sturm's sequence of polynomials starting with

$$P(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$$

and its derivative  $P'(x)$  as the first two polynomials in the sequence.

(ii) it uses the Sturm's sequence to count the number of roots within sub-intervals of  $x_1 \leq x \leq x_2$ , refining by bisection until brackets are found for each separate root.

(iii) having found brackets for the roots it then uses a combination of bisection and Newton-Raphson to locate each root. Bisection is used to stay within the bracket and the Newton provides the fast convergence when close to the root.

The Sturm's sequence method cannot always be guaranteed because it supposes that the  $k$ -th polynomial of the sequence is of degree  $n-k+1$ . Consequently if the coefficient of the highest power of  $x$  in the  $k$ -th polynomial is small or zero, large errors can result. It is strongly recommended that the error estimates returned in the array **E** are heeded as the subroutine does not take corrective action or provide any other indication of trouble.