1 SUMMARY

To find the **first m terms of the Taylor series expansions of** $S(x)=\sin\{A(x)\}$ **and** $C(x)=\cos\{A(x)\}$ such that

$$S'(x)\equiv A'(x)C(x), \qquad S(0)=\sin(a_1),$$

$$C'(x)\equiv -A'(x)S(x), \qquad C(0)=\cos(a_1).$$

Let

$$A(x)=a_1+a_2x+a_3x^2+...+a_{n+1}x^n$$

then the two Taylor series expansions of the form

$$S(x)=s_1+s_2x+s_3x^2+...+s_mx^{m-1}+...$$

and

$$C(x)=c_1+c_2x+c_3x^2+...+c_mx^{m-1}+...$$

are obtained by considering identities between $A(x)$, $S(x)$ and $C(x)$.

ATTRIBUTES — Version: 1.0.0. **Remark:** Formerly `PD02D` **Types:** `PD07A; PD07AD.` **Original date:** December 1970. **Origin:** M.J.Hopper, Harwell.

2 HOW TO USE THE PACKAGE

2.1 Argument list

The single precision version

```
CALL PD07A(A,N,S,C,M)
```

The double precision version

```
CALL PD07AD(A,N,S,C,M)
```

`A` is a `REAL` (`DOUBLE PRECISION` in the D version) array which must be set by the user to the coefficients of the polynomial $A(x)$, so that `A(j)` = a_j, j=1, 2,..., n+1. Note that the elements of `A` are temporarily modified by the subroutine to `(J-1)*A(J), J=2, 3, ..., N+1` but are restored to their original values before returning to the caller.

`N` is an `INTEGER` variable which must be set by the user to n the degree of the polynomial $A(x)$.

`S` is a `REAL` (`DOUBLE PRECISION` in the D version) array of length at least m in which the subroutine will return the first m terms of the expansion $S(x)$, i.e., `S(j)` = s_j, j=1, 2,..., m.

`C` is a `REAL` (`DOUBLE PRECISION` in the D version) array of length at least m in which the subroutine will return the first m terms of the expansion $C(x)$, i.e., `C(j)` = c_j, j=1, 2,..., m.

`M` is an `INTEGER` variable which must be set by the user to m the number of terms required from the two expansions $S(x)$ and $C(x)$.

3 GENERAL INFORMATION

Workspace: none.

Use of common: none.

Other routines called directly: none.

Input/output: none.

Restrictions: $n \geq 0$, $m \geq 0$.

4 METHOD

Assume $S(x) = \sin\{A(x)\}$ and $C(x) = \cos\{A(x)\}$. At x=0 these two expressions give $s_1 = \sin(a_1)$ and $c_1 = \cos(a_1)$. Differentiating the two expressions gives $S'(x) = A'(x)C(x)$ and $C'(x) = -A'(x)S(x)$. Then equating coefficients of like powers of x obtains the coupled recurrence relations

$$s_1 = \sin(a_1),$$

$$s_i = \frac{1}{(i-1)}\left[a_2 c_{i-1} + 2a_3 c_{i-2} + ... + (i-1)a_i c_1\right],$$

for i = 2, 3,..., m, and

$$c_1 = \cos(a_1),$$

$$c_i = \frac{-1}{(i-1)}\left[a_2 s_{i-1} + 2a_3 s_{i-2} + ... + (i-1)a_i s_1\right],$$

for i = 2, 3,..., m.