1 SUMMARY

To find the **first** m **terms of the Taylor series expansion of** $B(x)=\{A(x)\}^t$ such that $A(x)B'(x)\equiv tA'(x)B(x)$ and $B(0)=a_1^t$, Let

$$A(x)=a_1+a_2x+a_3x^2+...+a_{n+1}x^n,$$

where the first k coefficients a_i, i=1, 2,..., k can be zero with $a_{k+1}\neq 0$, then the Taylor series expansion

$$B(x)=x^s(b_1+b_2x+b_3x^2+...+b_mx^{m-1}+...$$

where $s=kt$, is obtained by considering identities between $A(x)$ and $B(x)$.

ATTRIBUTES — Version: 1.0.0. **Types:** `PD08A; PD08AD.` **Original date:** December 1970. **Origin:** M.J.Hopper, Harwell.

2 HOW TO USE THE PACKAGE

2.1 Argument list

The single precision version

```
CALL PD08A(A,N,B,M,T,S)
```

The double precision version

```
CALL PD08AD(A,N,B,M,T,S)
```

`A` is a `REAL` (`DOUBLE PRECISION` in the D version) array which must be set by the user to the coefficients of the polynomial $A(x)$, so that `A(j)` = a_j, j=1, 2,..., n+1. If the first k elements `A(J), J=1,K` are zero the subroutine will detect this (the value of $s=kt$ is returned in argument `S`).

`N` is an `INTEGER` variable which must be set by the user to n the degree of the polynomial $A(x)$.

`B` is a `REAL` (`DOUBLE PRECISION` in the D version) array of length at least m in which the subroutine will return the first m terms of the expansion $B(x)$, i.e., `B(j)` = b_j, j=1, 2,..., m.

`M` is an `INTEGER` variable which must be set by the user to m the number of terms required from the expansion $B(x)$.

`T` is a `REAL` (`DOUBLE PRECISION` in the D version) variable which must be set by the user to the power t to which the polynomial $A(x)$ is to be raised.

`S` is a `REAL` (`DOUBLE PRECISION` in the D version) variable which is set by the subroutine to the value of $s=kt$, where k is the number of leading coefficients of $A(x)$ found to be zero.

3 GENERAL INFORMATION

Workspace: none.

Use of common: none.

Other routines called directly: none.

Input/output: none.

Restrictions: $n\geq 0$, $m\geq 0$.

4 METHOD

Let $A(x) = x^k \tilde{A}(x)$ then

$$[x^k \tilde{A}(x)]^t = x^{kt} [\tilde{A}(x)]^t$$

and then letting $B(x) = x^{kt} \tilde{B}(x)$ it follows that

$$\tilde{B}(x) = [\tilde{A}(x)]^t \text{ with } b_1 = a_{k+1}^t \text{ at } x = 0, \ (a_{k+1} > 0).$$

Differentiating $\tilde{B}(x) = [\tilde{A}(x)]^t$ gives

$$\tilde{B}'(x) = t\tilde{A}'(x)[\tilde{A}(x)]^{t-1},$$

hence

$$\tilde{A}(x)\tilde{B}'(x) = t\tilde{A}'(x)\tilde{B}(x).$$

Equating coefficients of like powers of x then gives the reccurence relation

$$b_1 = a_{k+1}^t,$$

$$b_i = \frac{1}{(i-1)a_{k+1}} [(t-i+2)a_{k+2} b_{i-1} + (2t-i+3)a_{k+3} b_{i-2} + \ldots$$

$$+ \{(i-2)t-1\} a_{k+i-1} b_2 + (i-1)t a_{k+1} b_{1]},$$

for i=2, 3,..., m. Note: if $a_{k+1} < 0$ the polynomial $\hat{A}(x) = (-1)A(x)$ will satisfy the conditions required by the subroutine and then $[A(x)]^t = (-1)^t [\hat{A}(x)]^t$.