1 SUMMARY

To **evaluate a polynomial expressed as an expansion of Chebyshev polynomials,** i.e. given the polynomial

$$P(x)=\sum_{k=0}^{n} a_k T_k(x)$$

where the $T_k(x)$ k=0,1,...,n is defined over the interval $u \le x \le v$, the subroutine evaluates $P(x)$ for any x in $\{u,v\}$.

The Chebyshev polynomials are defined over the range $u \le x \le v$ by the three-term recurrence relation

$$T_0(x)=1,$$
$$T_1(x)=\frac{2x-u-v}{v-u},$$
$$T_k(x)=2\left(\frac{2x-u-v}{v-u}\right)T_{k-1}(x)-T_{k-2}(x) \qquad k=2,3,...,n$$

ATTRIBUTES — **Version:** 1.0.0. **Remark:** Designed for use with `PE12`. **Types:** `PE02A, PE02AD`. **Original date:** May 1980. **Origin:** C.Birch*, Harwell.

2 HOW TO USE THE PACKAGE

2.1 Argument list and calling sequence

Single precision version

```
VALUE=PE02A(A,N,X,U,V,IB)
```

Double precision version

```
VALUE=PE02AD(A,N,X,U,V,IB)
```

`A` is a `REAL` (`DOUBLE PRECISION` in the D version) array of length at least n+1 which must be set by the user to the coefficients of the Chebyshev expansion, i.e. the user must set `A(K), K=1,N+1` to the values of a_k, k=0,1,...,n. This argument is not altered by the subroutine.

`N` is an `INTEGER` variable which must be set by the user to n the degree of the polynomial. This argument is not altered by the subroutine.

`X` is a `REAL` (`DOUBLE PRECISION` in the D version) variable which must be set by the user to the value of x at which the value of the polynomial is to be evaluated. This argument is not altered by the subroutine.

Note that if the value of x is outside the range $\{u,v\}$ the value of the polynomial returned by the subroutine could be highly inaccurate.

`U` is a `REAL` (`DOUBLE PRECISION` in the D version) variable which must be set by the user to the value of u the lower limit of the range of x for which the Chebyshev polynomials are defined. Thsi argument is not altered by the subroutine.

`V` is a `REAL` (`DOUBLE PRECISION` in the D version) variable which must be set by the user to the value of v the upper limit of the range of x for which the Chebyshev polynomials are defined. This argument is not altered by the subroutine.

`IB` is an `INTEGER` variable which must be set by the user to indicate whether it is required to evaluate the Chebyshev expansion as an odd, even or mixed function (if in doubt indicate mixed). Set `IB` to one of the

following three values

1 for an odd function.
2 for an even function.
3 for a mixed function.

This argument is not altered by the subroutine.

`PE02A` and `PE02AD` are Fortran `FUNCTION` subroutines. The double precision version `PE02AD` must be declared `DOUBLE PRECISION` in the calling program if a double precision result is required.

3 GENERAL INFORMATION

Use of common: none.

Workspace: none.

Other routines called directly: none.

Input/output: none.

Restrictions:

$u \le x \le v$,
$u < v$,
$n \ge 0$.

4 METHOD

For the mixed function and limits $u \le x \le v$ we have

$$\sum_{i=0}^{n} a_i T_i(x) = \sum_{i=0}^{n-3} a_i T_i(x) + ...$$

$$...+ (a_{n-2} - a_n) T_{n-2}(x) + \left(2\frac{2x-u-v}{v-u} a_n + a_{n-1} \right) T_{n-1}(x)$$

and by continuing in a similar manner we obtain

$$\sum_{i=0}^{n} a_i T_i(x) = A_k T_1(x) + A_{k-1}$$

where A_k and A_{k-1} are obtained by a recurrence relation, see also Clenshaw (1955), M.T.A.C., **IX**, 118.