1 SUMMARY

Given a polynomial $f(x)$ (expressed as a sum of orthogonal polynomials), which is defined by

a) a set of coefficients a_0, a_1,..., a_m and

b) a set of recurrence parameters, α_i and β_i ($0 \le i \le m$) which define the orthogonal polynomials p_i which have been generated with respect to a set of x values

$$f(x)=a_0 p_0(x)+a_1 p_1(x)+...a_m p_m(x),$$

this subroutine evaluates $f(x^*)$ and optionally $f'(x^*)$ and $f''(x^*)$ for a given value of $x=x^*$

$$f'(x^*)=\frac{\partial}{\partial x}f(x^*)$$

$$f''(x^*)=\frac{\partial^2}{\partial x^2}f(x^*)$$

The orthogonal polynomials $p_i(x)$ are specified by the three term recurrence relationship as follows

$$p_0(x)=1$$

$$p_1(x)=x-\alpha_1$$

$$p_r(x)=(x-\alpha_r)p_{r-1}(x)-\beta_r p_{r-2}(x) \text{ in which}$$

$$\alpha_r=\sum_i(W_{yi}[p_{r-1}(x_i)]^2 x_i) \sum_i(W_{yi}[p_{r-1}(x_i)]^2)$$

$$\beta_r=\sum_i(W_{yi}[p_{r-1}(x_i)p_{r-2}(x_i)]^2) \sum_i(W_{yi}[p_{r-2}(x_i)]^2)$$

where the polynomials have been generated over the set of points x_i with respect to the weights W_{yi} (see also `VC01`).

ATTRIBUTES — **Version:** 1.0.0. **Types:** `PE09A`, `PE09AD`. **Original date:** May 1975. **Origin:** W.R. Owen, University of Queensland, Australia.

2 HOW TO USE THE PACKAGE

2.1 The argument list and calling sequence

The single precision version

```
CALL PE09A(MORDER,ALPHA,BETA,ACOEF,X,FAD,IDERSW)
```

The double precision version

```
CALL PE09AD(MORDER,ALPHA,BETA,ACOEF,X,FAD,IDERSW)
```

`MORDER` is an `INTEGER` giving the order (degree) of the polynomial to be evaluated.

`ALPHA` is a `REAL` (`DOUBLE PRECISION` in the D version) array containing the recurrence relationship parameter, such that

`ALPHA(K)` = α_k $0 \le$ `K` $\le$ `MORDER`, $0 \le k \le$ `MORDER`

`BETA` is a `REAL` (`DOUBLE PRECISION` in the D version) array containing the recurrence relationship parameters, such that

`BETA(K)` = β_k $1 \le$ `K` $\le$ `MORDER`, $1 \le k \le$ `MORDER`

`ACOEF` is a `REAL` (`DOUBLE PRECISION` in the D version) array containing the coefficients of the expansion of the polynomial such that

`ACOEF(K)` = a_{k-1} $1 \le$ `K` $\le$ `(MORDER+1)`, $1 \le k \le$ `(MORDER+1)`

`X` is a `REAL` (`DOUBLE PRECISION` in the D version) variable giving the x value for which $f(x)$ $f'(x)$, $f''(x)$ are required.

`FAD` is a `REAL` (`DOUBLE PRECISION` in the D version) array of length 3 which will be set on return, with `FAD(1)` = $f(x)$, `FAD(2)` = $f'(x)$, `FAD(3)` = $f''(x)$. This argument may be an array of length 2 or a scalar for `IDERSW=1` and `IDERSW=0` respectively.

`IDERSW` is an `INTEGER` acting as a switch indicating the results required

if `IDERSW` ≤ 0 a function value only will be returned

if `IDERSW` = 1 the function value and the first derivative will be returned

if `IDERSW` ≥ 2 the function, first and second derivative values will be returned.

3 GENERAL INFORMATION

Use of common: : None

Workspace: : None required

Input/output: : None

Restrictions: : None **Original date:** : May 1975

4 METHOD

The routine uses a recursive algorithm described by Smith, F.J., *Math. Comp.,* **19,** pp.33-36 (1965).