1 SUMMARY

Given n values of a function $f(x)$ **calculates the mth degree minimax polynomial** approximation $P(x)$, where $n \geq m$, and $2 \leq m \leq 25$, such that

$$\max_{1 \leq k \leq n} \left| f(x_k) - P(x_k) \right|$$

is minimized.

See P.C. Curtiss and W.L. Frank, Journal, A.C.M., 1959.

ATTRIBUTES — **Version:** 1.0.0. **Types:** PE11A; PE11AD. **Original date:** November 1966. **Origin:** S.Marlow, Harwell.

2 HOW TO USE THE PACKAGE

2.1 The argument list and calling sequence

The single precision version:

```
CALL PE11A(F,X,N,M,NE,I,A,NC)
```

The double precision version:

```
CALL PE11AD(F,X,N,M,NE,I,A,NC)
```

F is a REAL (DOUBLE PRECISION in the D version) array of length at least n, which must be set by the user to the function values so that F(j) = $f(x_j)$

X is a REAL (DOUBLE PRECISION in the D version) array of length at least n, which must be set by the user to the points x_j so that X(j)=x_j.

N is an INTEGER which must be set by the user to the number of points.

M is an INTEGER variable which must be set by the user to the maximum degree that the approximating polynomial $p^*(x)$ will be allowed to take.

NE is an INTEGER variable which is an error return parameter which is set to zero if M < N, and is set to 1 otherwise.

I is an INTEGER array of length at least M+4 which will be set by the subroutine to the position of the M+2 reference points so that the k th reference point is X(I(K+1)).

A is a REAL (DOUBLE PRECISION in the D version) array of length at least M+1 set by the subroutine to the coefficients of the polynomial $p^*(x)$ so that A(J+1) = a_{j+1} where

$$p^*(x) = \sum_{j=0}^{m} a_{j+1} x^j$$

NC is an INTEGER variable which controls output from the subroutine. The basic output from PE11A/AD is the array A defining the polynomial. If no other output is required then set NC=3.

If NC=1 or 2, then a table of the values x_i, $f(x_i)$, $p^*(x_i)$, $f(x_i) - p^*(x_i)$ is printed.

If NC=2 or 4, then the vector **u** is identified by printing the subscripts $i_1, i_2, \ldots, i_{m+2}$ where **u** = $(x_{i_1}, x_{i_2}, \ldots, x_{i_{m+2}})$. With this value of NC , δ is also printed.

If NC $\leq$ 0 then all printing is suppressed.

2.2 The common block.

A labelled common block `PE11B/BD` is used.

The single precision version

```
COMMON/PE11B/LP,LPD
```

The double precision version

```
COMMON/PE11BD/LP,LPD
```

`LP` is an `INTEGER` variable set to the stream number for printing of results. (default value is 6).

`LPD` is an `INTEGER` variable set to the stream number for diagnostic printing. (default value is 6).

3 GENERAL INFORMATION

Use of Common: a `COMMON` block `PE11B/BD` is used, see § 2.2.

Workspace: none.

Other subroutines: none

Input/Output: see § 2.1.

4 METHOD

The method used is described in P.C.Curtiss and W.L.Frank, Journal of A.C.M. 1959, pp.395-404. If **u** = $(u_{0,} u_{1,} ...., u_{m+1})$ where u_i is some point of the set $x_{1,} x_{2,} ..., x_n$ consider the equations

(i) $a_0 + a_1 u_k + a_2 u_k^2 + ... + a_n u_k^n + (-1)^k \delta = f(u_k)$; k=1,2,..

(ii) $|\ \delta\ | = \max_{1 \le i \le n} | f(x_i) - (a_0 + a_1 x_i + ... + a_n x_i^n)|$

Then (i) can be solved for $a_{0,} a_{1,} ..., a_{n,} \delta$. There is a **u** such that the solution of (i) satisfies (ii). If $q(x)$ is the polynomial with coefficients $a_{0,} a_{1,} ... a_n$ then $p^*(x) = q(x)$. Note that **u** is not necessarily unique.

5 EXAMPLE OF USE

Suppose we have a table of points with corresponding function values and wish to find an approximating polynomial to the function. Then code of the following form could be used.

```
      DOUBLE PRECISION F(20),X(20),A(10)
      INTEGER II(10)
      DATA F( 1)/ 0.98322D+04/,F( 2)/ 0.59127D+04/,F( 3)/ 0.32993D+04/,
     *     F( 4)/ 0.16679D+04/,F( 5)/ 0.73080D+03/,F( 6)/ 0.25775D+03/,
     *     F( 7)/ 0.62912D+02/,F( 8)/ 0.72500D+01/,F( 9)/-0.98339D+00/,
     *     F(10)/-0.27854D+01/,F(11)/ 0.90000D+01/,F(12)/ 0.88816D+02/,
     *     F(13)/ 0.33801D+03/,F(14)/ 0.90512D+03/,F(15)/ 0.19900D+04/,
     *     F(16)/ 0.38360D+04/,F(17)/ 0.67405D+04/,F(18)/ 0.11043D+05/,
     *     F(19)/ 0.17136D+05/,F(20)/ 0.25456D+05/
      DATA X( 1)/-0.85000D+01/,X( 2)/-0.75000D+01/,X( 3)/-0.65000D+01/,
     *     X( 4)/-0.55000D+01/,X( 5)/-0.45000D+01/,X( 6)/-0.35000D+01/,
     *     X( 7)/-0.25000D+01/,X( 8)/-0.15000D+01/,X( 9)/-0.50000D+00/,
     *     X(10)/ 0.50000D+00/,X(11)/ 0.15000D+01/,X(12)/ 0.25000D+01/,
     *     X(13)/ 0.35000D+01/,X(14)/ 0.45000D+01/,X(15)/ 0.55000D+01/,
     *     X(16)/ 0.65000D+01/,X(17)/ 0.75000D+01/,X(18)/ 0.85000D+01/,
     *     X(19)/ 0.95000D+01/,X(20)/ 0.10500D+02/
C      NUMBER OF FUNCTION VALUES.
      N=20
```

```
C      MAXIMUM DEGREE OF APPROXIMATING POLYNOMIAL.
      M=4
C      PRINT TABLE OF VALUES.
      NC=1
      CALL PE11AD(F,X,N,M,NE,II,A,NC)
      WRITE(6,20)(A(I),I=1,5)
   20 FORMAT(' THE COEFFICIENTS OF THE POLYMONIAL ',/,
     +       ' A(1)+A(2)*X+A(3)*X**2+A(4)*X**3+A(5)*X**4 ARE ',/,
     +       ' A(1)=',D14.6,' A(2)=',D14.6,' A(3)=',D14.6,/,
     +       ' A(4)=',D14.6,' A(5)=',D14.6)
      STOP
      END
```

The following output would be produced.

```
  I        X              F(X )           P(X )         F(X )-P(X )
            I                I               I              I     I

   1  -0.850000E+01   0.98322000E+04   0.98324707E+04  -0.27073554E+00
   2  -0.750000E+01   0.59127000E+04   0.59117070E+04   0.99297924E+00
   3  -0.650000E+01   0.32993000E+04   0.32999579E+04  -0.65797720E+00
   4  -0.550000E+01   0.16679000E+04   0.16672174E+04   0.68255098E+00
   5  -0.450000E+01   0.73080000E+03   0.73148041E+03  -0.68041310E+00


   6  -0.350000E+01   0.25775000E+03   0.25874297E+03  -0.99297924E+00
   7  -0.250000E+01   0.62912000E+02   0.63002390E+02  -0.90389996E-01
   8  -0.150000E+01   0.72500000E+01   0.62570207E+01   0.99297924E+00
   9  -0.500000E+00  -0.98339000E+00  -0.14936202E+01   0.51023026E+00
  10   0.500000E+00  -0.27854000E+01  -0.22488920E+01  -0.53650794E+00


  11   0.150000E+01   0.90000000E+01   0.99929792E+01  -0.99297924E+00
  12   0.250000E+01   0.88816000E+02   0.89234900E+02  -0.41890028E+00
  13   0.350000E+01   0.33801000E+03   0.33748091E+03   0.52908944E+00
  14   0.450000E+01   0.90512000E+03   0.90473618E+03   0.38381766E+00
  15   0.550000E+01   0.19900000E+04   0.19890070E+04   0.99297924E+00


  16   0.650000E+01   0.38360000E+04   0.38363008E+04  -0.30086374E+00
  17   0.750000E+01   0.67405000E+04   0.67406262E+04  -0.12628205E+00
  18   0.850000E+01   0.11043000E+05   0.11043992E+05  -0.99297924E+00
  19   0.950000E+01   0.17136000E+05   0.17136411E+05  -0.41179170E+00
  20   0.105000E+02   0.25456000E+05   0.25455894E+05   0.10531135E+00


 THE COEFFICIENTS OF THE POLYMONIAL
 A(1)+A(2)*X+A(3)*X**2+A(4)*X**3+A(5)*X**4 ARE
 A(1)= -0.199576D+01 A(2)= -0.100535D+01 A(3)= -0.198992D-02
 A(4)=  0.100030D+01 A(5)=  0.200005D+01
```