1 SUMMARY

Given a polynomial $P(x)$ of degree n, expressed as an expansion in **orthogonal polynomials, obtains the expansion in terms of Chebyshev polynomials,** i.e. given that

$$P(x)=\sum_{k=0}^{n} c_k Q_k(x)$$

it finds the coefficients a_k k=0,1,...,n such that

$$P(x)=\sum_{k=0}^{n} a_k T_k(x)$$

The orthogonal polynomials $Q_k(x)$ k=0,1,...,n are defined by the three-term recurrence relation

$$Q_0(x)=1,$$
$$Q_1(x)=x-\alpha_1,$$
$$Q_k(x)=(x-\alpha_k)Q_{k-1}(x)-\beta_k Q_{k-2}(x) \qquad k=2,3,...$$

and where the Chebyshev polynomials $T_k(x)$ k=0,1,...,n are defined by the three-term recurrence relation

$$T_0(x)=1,$$
$$T_1(x)=\frac{(2x-u-v)}{(v-u)},$$
$$T_k(x)=\frac{2(2x-u-v)}{(v-u)}T_{k-1}(x)-T_{k-2}(x)$$

for the limits $u \le x \le v$.

ATTRIBUTES — **Version:** 1.0.0. **Types:** `PE12A`, `PE12AD`. **Original date:** May 1980. **Origin:** C.Birch*, Harwell.

2 HOW TO USE THE PACKAGE

2.1 Argument list and calling sequence

The single precision version

```
CALL PE12A(ALPHA,BETA,C,A,WORK1,WORK2,N,U,V)
```

The double precision version

```
CALL PE12AD(ALPHA,BETA,C,A,WORK1,WORK2,N,U,V)
```

`ALPHA` is a `REAL` (`DOUBLE PRECISION` in the D version) array of length at least n which must be set by the user to the values α_k k=1,2,...,n. This argument is not altered by the subroutine.

`BETA` is a `REAL` (`DOUBLE PRECISION` in the D version) array of length at least n which must be set by the user to the values β_k k=2,3,...,n. Note that the first element `BETA(1)` is not used. This argument is not altered by the subroutine.

`C` is a `REAL` (`DOUBLE PRECISION` in the D version) array of length at least n+1 which must be set by the user to contain the orthogonal expansion coefficients c_k k=0,1,...,n and these must be stored in `C(K), K=1,N+1.` This argument is not altered by the subroutine.

`A` is a `REAL` (`DOUBLE PRECISION` in the D version) array of length at least n+1 in which the subroutine will place the values of the coefficients of the Chebyshev expansion a_k k=0,1,...,n. These will be stored in `A(K), K=1,N+1.`

`WORK1` is a `REAL` (`DOUBLE PRECISION` in the D version) array of length at least n+1 which is used by the subroutine as workspace.

`WORK2` is a `REAL` (`DOUBLE PRECISION` in the D version) array of length at least n+2 which is used by the subroutine as workspace.

`N` is an `INTEGER` variable which must be set by the user to the value of n the degree of the polynomial.

`U` is a `REAL` (`DOUBLE PRECISION` in the D version) variable which must be set by the user to u the lower limit of the range of x for the Chebyshev expansion.

`V` is a `REAL` (`DOUBLE PRECISION` in the D version) variable which must be set by the user to v the upper limit for the range of x for the Chebyshev expansion.

3 GENERAL INFORMATION

Use of common: none.

Workspace: provided by the user, 2n+3 words.

Other subroutines: none.

Input/Output: none.

Restrictions:

$u < v$,

$n \geq 0$.

4 METHOD

We are given that

$$P(x) \equiv c_0 Q_0(x) + c_1 Q_1(x) + ... + c_n Q_n(x)$$

and we require

$$P(x) \equiv a_0 T_0(x) + a_1 T_1(x) + ... + a_n T_n(x)$$

Let $Q_j(x) \equiv {\sum_{i=0}^{j}}' b_{i,j} T_i(x)$ (where $\sum'$ signifies that the first term should be halved) and this means that

$$a_j = \sum_{k=j}^{n} c_k b_{j,k} \quad j=1,2,...,n$$

starting with $a_0 = c_0$.

We have that

$$Q_k(x) \equiv (x - \alpha_k) Q_{k-1}(x) - \beta_k Q_{k-2}(x)$$

so given that the limits for x are –1 to 1

$$Q_k(x) \equiv \sum_{i=0}^{k-1}{}' b_{i,k-1} \left(\tfrac{1}{2}T_{i+1}(x) + \tfrac{1}{2}T_{i-1}(x) - \alpha_k T_i(x)\right) - \ldots$$
$$\ldots - \sum_{i=0}^{k-2}{}' \beta_k b_{i,k-2} T_i(x)$$

Therefore

$$b_{0,0} = 2$$
$$b_{0,k+1} = b_{1,k} - \alpha_k b_{0,k} - \beta_k b_{0,k-1}$$
$$b_{i,k+1} = \tfrac{1}{2}(b_{i-1,k} + b_{i+1,k}) - \alpha_k b_{i,k} - \beta_k b_{i,k-1}$$

and $b_{i,k}$ such that $i < k$, i or k is less than zero, or $k > n$ can be assumed to be equal to zero.

If the limits on x are $u \le x \le v$ the more general form of the recurrence relation can be used.

$$b_{0,0} = 2,$$
$$b_{0,k+1} = \tfrac{1}{2}(v-u) b_{1,k+1} + (\tfrac{1}{2}(v+u) - \alpha_k) b_{0,k} - \beta_k b_{0,k-1},$$
$$b_{i,k+1} = \tfrac{1}{4}(v-u)(b_{i+1,k} + b_{i-1,k}) + \ldots$$
$$\ldots + (\tfrac{1}{2}(u+v) - \alpha_k) b_{i,k} - \beta_k b_{i,k-1}$$