1 SUMMARY

To **integrate a cubic spline** $S(x)$ between **limits which are knot points,** i.e. given knots ξ_i, function values $S_i = S(\xi_i)$ and derivative values $g_i = S'(x_i)$, i=1,2,...,n ($n \geq 2$) evaluates the integral

$$\int_{\xi_j}^{\xi_k} S(x)\,dx$$

where ξ_j and ξ_k are two knot points of $S(x)$.

ATTRIBUTES — **Version:** 1.0.0. **Remark:** see `QG02` for when the limits are not knot points. **Types:** `QG01A`; `QG01AD`. **Original date:** March 1974. **Origin:** M.J.Hopper, Harwell.

2 HOW TO USE THE PACKAGE

2.1 Argument list and calling sequence

The single precision version

```
Q=QG01A(J,K,N,XI,S,G)
```

The double precision version

```
DOUBLE PRECISION Q
  -        -
Q=QG01AD(J,K,N,XI,S,G)
```

The arguments

`J` is an `INTEGER` variable which must be set by the user to specify which knot point is to be used as the lower limit of the integration. See next argument.

`K` is an `INTEGER` variable which must be set by the user to specify which knot point is to be used as the upper limit of the integration.

If either `J` or `K` is outside the range of 1 to n the integral is evaluated on the assumption that $S(x)=0$ for $x<\xi_1$ or $x>\xi_n$. If `J` > `K` the sign of the integral is reversed.

`N` is an `INTEGER` variable which must be set by the user to n the number of knot points. **Restriction:** $n \geq 2$.

`XI` is a `REAL` (`DOUBLE PRECISION` in the D version) array of length at least n which must be set by the user to the knot values ξ_i i=1, 2,..., n. The knots must be ordered so that $\xi_1 \leq \xi_2 \leq ... \leq \xi_n$.

`S` is a `REAL` (`DOUBLE PRECISION` in the D version) array of length at least n which must be set by the user to the spline values $S_i = S(\xi_i)$ i=1, 2,..., n.

`G` is a `REAL` (`DOUBLE PRECISION` in the D version) array of length at least n which must be set by the user to the first derivative values of the spline at the knots, i.e. set to $g_i = S'(\xi_i)$ i=1, 2,..., n.

Function value

`QG01A` and `QG01AD` are Fortran `FUNCTION` subroutines and will be set to the value of the integral on return.

3 GENERAL INFORMATION

Use of common: none.

Workspace: none.

Other routines called directly: none.

Input/output: none.

Restrictions: $n \geq 2$, $\xi_1 \leq \xi_2 \leq ... \leq \xi_n$.

4 METHOD

Let the knots be

$$\xi_i, \quad i=1, 2,..., n,$$

the spline values be

$$S_i = S(\xi_i) \quad i=1, 2,..., n,$$

and the first derivative values be

$$g_i = \left.\frac{dS(x)}{dx}\right|_{x=\xi_i} \quad i=1, 2,..., n;$$

then the integration over one knot interval, the i-th say, is simply

$$Q_i = \frac{h}{2}\{S_{i+1}+S_i\} - \frac{h^2}{12}\{g_{i+1}-g_i\}$$

where $h = \xi_{k+1} - \xi_k$.

The integral over ξ_j to ξ_k is obtained by accumulating the integrals over the knot intervals in j to k. The subroutine first makes sure that j and k are sensible.