1 SUMMARY

To integrate a cubic spline \( S(x) \) between limits which are knot points, i.e. given knots \( \xi_i \), function values \( S_i = S(\xi_i) \) and derivative values \( g_i = S'(\xi_i) \), \( i=1,2,...,n \) \( (n \geq 2) \) evaluates the integral

\[
\int_{\xi_j}^{\xi_k} S(x) \, dx
\]

where \( \xi_j \) and \( \xi_k \) are two knot points of \( S(x) \).

**ATTRIBUTES** — **Version**: 1.0.0. **Remark**: see QG02 for when the limits are not knot points. **Types**: QG01A; QG01AD. **Original date**: March 1974. **Origin**: M.J.Hopper, Harwell.

2 HOW TO USE THE PACKAGE

2.1 Argument list and calling sequence

*The single precision version*

\[ Q = \text{QG01A}(J, K, N, XI, S, G) \]

*The double precision version*

\[
\text{DOUBLE PRECISION } Q \\
Q = \text{QG01AD}(J, K, N, XI, S, G)
\]

The arguments

\( J \) is an INTEGER variable which must be set by the user to specify which knot point is to be used as the lower limit of the integration. See next argument.

\( K \) is an INTEGER variable which must be set by the user to specify which knot point is to be used as the upper limit of the integration.

If either \( J \) or \( K \) is outside the range of 1 to \( n \) the integral is evaluated on the assumption that \( S(x) = 0 \) for \( x < \xi_1 \) or \( x > \xi_n \). If \( J > K \) the sign of the integral is reversed.

\( N \) is an INTEGER variable which must be set by the user to \( n \) the number of knot points. **Restriction**: \( n \geq 2 \).

\( XI \) is a REAL (DOUBLE PRECISION in the D version) array of length at least \( n \) which must be set by the user to the knot values \( \xi_i \), \( i=1,2,...,n \). The knots must be ordered so that \( \xi_1 \leq \xi_2 \leq ... \leq \xi_n \).

\( S \) is a REAL (DOUBLE PRECISION in the D version) array of length at least \( n \) which must be set by the user to the spline values \( S_i = S(\xi_i) \), \( i=1,2,...,n \).

\( G \) is a REAL (DOUBLE PRECISION in the D version) array of length at least \( n \) which must be set by the user to the first derivative values of the spline at the knots, i.e. set to \( g_i = S'(\xi_i) \), \( i=1,2,...,n \).

Function value

QG01A and QG01AD are Fortran FUNCTION subroutines and will be set to the value of the integral on return.
3 GENERAL INFORMATION

Use of common: none.
Workspace: none.
Other routines called directly: none.
Input/output: none.
Restrictions: \( n \geq 2, \xi_1 \leq \xi_2 \leq \ldots \leq \xi_n \).

4 METHOD

Let the knots be
\[ \xi_i, \quad i = 1, 2, \ldots, n, \]
the spline values be
\[ S_i = S(\xi_i), \quad i = 1, 2, \ldots, n, \]
and the first derivative values be
\[ g_i = \frac{dS(x)}{dx} \bigg|_{x=\xi_i}, \quad i = 1, 2, \ldots, n; \]
then the integration over one knot interval, the \( i \)-th say, is simply
\[ Q_i = \frac{h}{2} \{ S_{i+1} + S_i \} - \frac{h^2}{12} \{ g_{i+1} - g_i \} \]
where \( h = \xi_{i+1} - \xi_i \).

The integral over \( \xi_j \) to \( \xi_k \) is obtained by accumulating the integrals over the knot intervals in \( j \) to \( k \). The subroutine first makes sure that \( j \) and \( k \) are sensible.