1 SUMMARY

To evaluate the one-sided cumulative distribution function of **Student's** *t* **distribution** with *n* degrees of freedom, i.e. evaluate

$$P(n,t) = \frac{1}{B(\frac{1}{2},\frac{n}{2})n^{\frac{1}{2}}} \int_{-\infty}^{t} \left\{1+\frac{\theta^2}{n}\right\}^{-\frac{n+1}{2}} d\theta \quad -\infty \le t \le \infty$$

where *n* is positive.

ATTRIBUTES — Version: 1.0.0. **Types:** `SA02A`; `SA02AD`. **Original date:** December 1970. **Origin:** D.G.Papworth, MRC, Harwell.

2 HOW TO USE THE PACKAGE

2.1 Argument list

The single precision version

```
CALL SA02A(T,N,P)
```

The double precision version

```
CALL SA02AD(T,N,P)
```

`T` is a `REAL` (`DOUBLE PRECISION` in the D version) variable which must be set by the user to the *t* value.

`N` is an `INTEGER` variable which must be set by the user to *n* the number of degrees of freedom. **Restriction:** $n > 0$.

`P` is a `REAL` (`DOUBLE PRECISION` in the D version) variable set by the subroutine to the value of the cumulative function at *t* with *n* degrees of freedom, i.e. if a statistic *x* has a normal distribution with mean zero and unit variance, and χ^2 a random variable distributed as chi-squared with *n* degrees of freedom, then P is the probability of a statistic

$$x\sqrt{\frac{n}{\chi^2}} \le t.$$

For a two sides test, i.e. the probability of

$$\left| x\sqrt{\frac{n}{\chi^2}} \right| \le |t|$$

take `2*P-1`.

3 GENERAL INFORMATION

Use of common: none.

Workspace: none.

Other routines called directly: none.

Input/output: none.

4 METHOD

A series expansion is used, let $\alpha = \tan^{-1}\frac{t}{\sqrt{n}}$ then if n is even

$$P(n,t) = \frac{1}{2} + \frac{1}{2}\sin\alpha\left\{1 + \frac{1}{2}\cos^2\alpha + \frac{1.3}{2.4}\cos^4\alpha + \ldots + \frac{1.3.5\ldots(n-3)}{2.4.6\ldots(n-2)}\cos^{n-2}\alpha\right\}$$

If $n = 1$,

$$P(1,t) = \frac{1}{\pi}\left(\alpha + \frac{\pi}{2}\right)$$

and if n is odd and $n > 1$

$$P(n,t) = \frac{1}{2} + \frac{\alpha}{\pi} + \frac{1}{\pi}\sin\alpha\left\{\cos\alpha + \frac{2}{3}\cos^3\alpha + \frac{2.4}{3.5}\cos^5\alpha + \ldots + \frac{2.4\ldots(n-3)}{3.5\ldots(n-2)}\cos^{n-2}\alpha\right\}$$