1 SUMMARY

This subroutine finds a periodic cubic spline $S(x)$ that interpolates $n$ function values. The user supplies the function values $f_1, f_2, \ldots, f_n$ (with $f_1 = f_n$) at points $x_1 < x_2 < \ldots < x_n$ (which need not be equally spaced) and the subroutine finds derivative values $S'(x_i)$ at the points $x_i$. $S(x)$ is cubic on each interval $(x_i, x_{i+1})$, $i=1,2,\ldots,n-1$, is continuous with two continuous derivatives at the points $x_i$, $i=2,3,\ldots,n-1$ and $S'(x_1) = S'(x_n)$, $S''(x_1) = S''(x_n)$. The subroutine TG01 can be used to find values of $S(x)$.

ATTRIBUTES  
Version: 1.0.0.  
Types: TB15A, TB15AD.  
Calls: None.  
Original date: April 1985.  
72 lines.  
Origin: J.K. Reid, Harwell.  
Remark: This is a slightly rewritten version of TB05 and supersedes it.

2 HOW TO USE THE PACKAGE

2.1 Argument list

The single precision version

CALL TB15A(N,X,F,D,WORK,LP)

The double precision version

CALL TB15AD(N,X,F,D,WORK,LP)

$N$ is an INTEGER variable which must be set by the user to $n$, the number of data points. It is not altered.  
Restriction: $n \geq 4$.

$X$ is a REAL (DOUBLE PRECISION in the D version) array of length at least $n$ which the user must set to the points $x_i, i=1,2,\ldots,n$. It is not altered.  
Restriction: $x_1 < x_2 < \ldots < x_n$.

$F$ is a REAL (DOUBLE PRECISION in the D version) array of length at least $n$ which the user must set to the function values $f_1, f_2, \ldots, f_n$. It is not altered.  
Restriction: $f_1 = f_n$.

$D$ is a REAL (DOUBLE PRECISION in the D version) array of length at least $n$ which need not be set by the user. The subroutine puts the values of the first derivatives of the spline $S(x)$ at the points $x_i$ in $D(i), i=1,2,\ldots,n$.

$WORK$ is a REAL (DOUBLE PRECISION in the D version) array of length at least $3n$ which is used as a workspace. $W(1)$ is set to zero on a successful return and to a nonzero value on a failure. The subroutine prints a diagnostic message when $n < 4$, a point $x_i$ is out of order (see $X$) or $f_i \neq f_n$ (see $F$). $W(1)$ is set to 1, 2, 3 respectively.

$LP$ is an INTEGER variable which must be set by the user to a unit number for printing or to a non-positive number if printing is to be suppressed.

3 GENERAL INFORMATION

Use of common: None.

Other routines called directly: None.

Input/output: Output is under the control of argument $LP$.

Restrictions: $n \geq 4, x_1 < x_2 < \ldots < x_n, f_1 = f_n$
4 METHOD

The continuity of the second derivative $S''(x)$ at $x_i$, $i=2,3,\ldots,n-1$, can be expressed as a linear equation involving $S''(x_{i-1})$, $S''(x_i)$ and $S''(x_{i+1})$. Similarly the continuity of $S'(x)$ at $x_i$ can be expressed as a linear equation involving $S'(x_{i-1})$, $S'(x_i)$ and $S'(x_{i+1})$. Thus the unknown derivatives $S''(x_i)$, $i=1,2,\ldots,n-1$, satisfy a set of $n-1$ linear equations whose matrix is tridiagonal except for entries in positions $(1,n-1)$ and $(n-1,1)$. This set of equations is set up and solved by Gaussian elimination. No interchanges are needed because the matrix is diagonally dominant.

5 EXAMPLE OF USE

As a very simple example the following code finds a periodic cubic spline that interpolates the function values for the given data.

```fortran
DOUBLE PRECISION X(9),F(9),D(9),W(27)
DATA X/0.0000D0,2.0000D0,4.0000D0,6.0000D0,8.0000D0,
* 1.0000D1,1.2000D1,1.4000D1,1.6000D1/
DATA F/0.0000D0,1.0000D0,2.0000D0,4.0000D0,5.0000D0,
* 4.0000D0,3.0000D0,2.0000D0,0.0000D0/
DATA N/9/
DATA LP/6/
CALL TB05AD(N,X,F,D,W,LP)
WRITE(6,10)
10 FORMAT(10X,'X(I)',10X,'F(I)',10X,'D(I)')
WRITE(6,20)(X(I),F(I),D(I),I=1,N)
20 FORMAT(/(3F16.4))
STOP
END
```

This produces the following output

<table>
<thead>
<tr>
<th>X(I)</th>
<th>F(I)</th>
<th>D(I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0000</td>
<td>0.0000</td>
<td>-0.2946</td>
</tr>
<tr>
<td>2.0000</td>
<td>1.0000</td>
<td>0.6429</td>
</tr>
<tr>
<td>4.0000</td>
<td>2.0000</td>
<td>0.7232</td>
</tr>
<tr>
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<td>4.0000</td>
<td>0.9643</td>
</tr>
<tr>
<td>8.0000</td>
<td>5.0000</td>
<td>-0.0804</td>
</tr>
<tr>
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<td>-0.6429</td>
</tr>
<tr>
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<td>3.0000</td>
<td>-0.3482</td>
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<tr>
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<td>2.0000</td>
<td>-0.9643</td>
</tr>
<tr>
<td>16.0000</td>
<td>0.0000</td>
<td>-0.2946</td>
</tr>
</tbody>
</table>

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