



1 SUMMARY

If y_1, y_2, \dots, y_n are a set of observations of a function $f(x, v_1, v_2, \dots, v_m)$ at discrete values x_1, x_2, \dots, x_n , then VB01A finds the value of \mathbf{v} which minimizes

$$S = \sum_{i=1}^n w_i (y_i - f(x_i, \mathbf{v}))^2$$

where w_1, w_2, \dots, w_n are weights provided by the user. The method used is a modified Marquardt routine (see R. Fletcher, 'A modified Marquardt subroutine for nonlinear least squares', AERE-R 6799, 1971).

The user must provide initial estimates of the parameters v_j $j=1, 2, \dots, m$ and a subroutine to calculate values of the functions $f(x, \mathbf{v})$ and its derivatives with respect to the parameters $\partial f / \partial v_j$, $j=1, 2, \dots, m$.

The routine returns to the calling program

- (1) The final values of the v_i .
- (2) The standard deviations of the v_i .
- (3) The variance covariance matrix.
- (4) The fitted values $f(x_i, \mathbf{v})$.

It also prints out a history of the iterative process and full details of the solution and its statistical properties, including standard deviations and a chi-squared test on the variance ratio.

ATTRIBUTES — **Version:** 1.0.0. **Types:** VB01A, VB01AD. **Calls:** MA10, _DOT, OA01, SA01 and DERIV (a user subroutine). **Original date:** June 1972. **Origin:** R. Fletcher, Harwell.

2 HOW TO USE THE PACKAGE

2.1 The argument list and calling sequence

The single precision version

```
CALL VB01A(X, Y, W, Z, N, V, E, M, A, IA, MAXFN)
```

The double precision version

```
CALL VB01AD(X, Y, W, Z, N, V, E, M, A, IA, MAXFN)
```

- X is a REAL (DOUBLE PRECISION in the D version) array of n elements to be set by the user to the data points x_1, x_2, \dots, x_n .
- Y is a REAL (DOUBLE PRECISION in the D version) array of n elements to be set by the user to the observations y_1, y_2, \dots, y_n .
- W is a REAL (DOUBLE PRECISION in the D version) array of n elements to be set by the user to the weights w_1, w_2, \dots, w_n .
- Z is a REAL (DOUBLE PRECISION in the D version) array of n elements in which the fitted estimates $f(x_i, \mathbf{v})$ $i=1, 2, \dots, n$ are returned.
- N is an INTEGER to be set to n the number of observations.
- V is a REAL (DOUBLE PRECISION in the D version) array of m elements or more in which an initial estimate of v_1, v_2, \dots, v_m must be set. The optimum value of \mathbf{v} is returned on exit. In addition, $V(M+1), V(M+2), \dots$ may be used to communicate constants between the calling routine and the user subroutine DERIV.

- E** is a REAL (DOUBLE PRECISION in the D version) array of m elements in which the standard deviations of the v_i are returned on exit.
- M** is an INTEGER to be set to m the number of variables; $2 \leq m \leq 25$.
- A** is a REAL (DOUBLE PRECISION in the D version) array of two dimensions, at least m^2 , in which the variance-covariance matrix is returned on exit.
- IA** is an INTEGER set to the first dimension of **A**, i.e. the dimension statement in the calling program will have the form `A(IA,)`.
- MAXFN** is an INTEGER set to the maximum number of times which the sum of squares can be evaluated.

3 THE USER SUBROUTINE

The user must provide a subroutine headed

```
SUBROUTINE DERIV(X,V,F, FUNC, IFL)
REAL F(*), V(*)
```

which is passed a value x in **X** (one of the x_i) and a vector \mathbf{v} in **V**. The subroutine should calculate the value of $f(x, v_1, v_2, \dots, v_m)$ and return it in **FUNC** and calculate the values $\partial f / \partial v_i$ $i=1, 2, \dots, m$ and return them in **F(I) = I=1, M**. If the vector \mathbf{v} is unsatisfactory in any way (for instance **V(I)** may be negative when it is known to be positive from physical considerations) then **IFL** should be set equal to 1 and a **RETURN** made without setting **FUNC** or **F**. Finally constants may be communicated to **DERIV** from the calling routine by using **V(M+1), V(M+2), ...**

4 PRINTING

Printing occurs on stream 6 (line printer) although this can be changed (to stream 8 say) by including the statements

```
COMMON/VB01B/IP
IP=8
```

in the calling program. The print out commences with a history of each iteration of the iterative process given in the form

<i>iteration number</i>	<i>number of evaluations of S</i>
S	
V(1), V(2), . . . , V(M)	(8 to a line)
G(1), G(2), . . . , G(M)	(8 to a line)

where $2 * G(I)$ is the derivative $\partial S / \partial V(I)$. The fact that $G(I) \rightarrow 0$ for all **I** is an indication that the iteration has converged. Once the iteration has converged (the criterion being that the change in each **V(I)** is less than one tenth of its standard deviation) then the following information is printed out: the optimum **V(I)** and their standard deviations, the variance covariance matrix, the values x_i, y_i and $f(x_i, \mathbf{v})$ for each i , and finally the weighted sum of squares **S**, the number of degrees of freedom **N-M**, the variance ratio, and the probability of chance occurrence of a variance ratio as least as large as that obtained.

5. GENERAL

Use of common: A **COMMON** area named **VB01B/BD** is used. This need only be referenced if the user requires the output from **VB01A/AD** to be written on an output stream other than stream 6, see section 4.

Workspace: 800 words local to **VB01A/AD**.

Input/output: no input, for output see section 4.

Restrictions: $2 \leq m \leq 25$.