

## 1 SUMMARY

To solve the **two point boundary value** problem for the **second order linear** differential equation

$$y'' + f(x)y' + g(x)y = r(x) \quad x_1 \leq x \leq x_n$$

given boundary conditions of the form  $ay' + by = c$  at the two points  $x_1$  and  $x_n$ .

A finite difference approximation is used, see L. Fox, Proc. Roy. Soc. A.190, 1947. Initially 3rd and higher differences are ignored; then successive approximations are obtained by applying correction terms based upon 3rd and 4th differences.

The user must store in three arrays values of  $f(x)$ ,  $g(x)$  and  $r(x)$  tabulated at equal intervals over the range  $x_1 \leq x \leq x_n$  and it is the responsibility of the user to choose a tabulation interval which is sufficiently small that the finite difference approximations converge. The method is iterative and limits may be put on the number and size of the corrections made at each iteration. Error returns are made if the accuracy requirements are not met.

**ATTRIBUTES** — **Version:** 1.0.0. **Types:** DD11A; DD11AD. **Calls:** MA35 and TA03. **Original date:** May 1965. **Origin:** P.Hallowell, Atlas Lab., Chilton, Oxon. **Remark:** DD11 is a version of DD01 with the workspace passed as an argument.

## 2 HOW TO USE THE PACKAGE

### 2.1 Argument list and calling sequence

*The single precision version:*

```
CALL DD11A(Y,X1,XN,N,Y1,YN,F,G,R,E,LIM,EPS,K,W)
```

*The double precision version:*

```
CALL DD11AD(Y,X1,XN,N,Y1,YN,F,G,R,E,LIM,EPS,K,W)
```

**Y** is a REAL (DOUBLE PRECISION in the D version) array of length at least  $n$  in which the subroutine will return the final approximations  $y_i$   $i=1,2,\dots,n$  to the solution  $y(x_i)$   $i=1,2,\dots,n$ .

**X1, XN** are REAL (DOUBLE PRECISION in the D version) variables and must be set by the user to  $x_1$  and  $x_n$  the lower and upper limits of the range of  $x$  over which the equation is to be integrated.

**N** is an INTEGER variable and must be set by the user to  $n$  the number of equally spaced points  $x_i$   $i=1,2,\dots,n$  to be used to cover the range of integration. **Restriction:**  $n \geq 3$ .

**Y1, YN** are two REAL (DOUBLE PRECISION in the D version) arrays of length 3 in which the user must specify the boundary conditions at  $x_1$  and  $x_n$  respectively, i.e. if these are

$$a_1 y'_1 + b_1 y_1 = c_1$$

$y'_1$  and  $y_1$  to be evaluated at  $x_1$ , and

$$a_n y'_n + b_n y_n = c_n$$

$y'_n$  and  $y_n$  to be evaluated at  $x_n$ , then the user must set  $Y1(1) = a_1$ ,  $Y1(2) = b_1$ ,  $Y1(3) = c_1$  and similarly for **YN** and  $a_n$ ,  $b_n$  and  $c_n$ .

**F** is a REAL (DOUBLE PRECISION in the D version) array and the user must set the first  $n$  elements to  $f(x_i)$   $i=1,2,\dots,n$ .

**G** is a REAL (DOUBLE PRECISION in the D version) array and the user must set the first  $n$  elements to  $g(x_i)$

$i=1,2,\dots,n$ .

- R is a REAL (DOUBLE PRECISION in the D version) array and the user must set the first  $n$  elements to  $r(x_i)$   $i=1,2,\dots,n$ .
- E is a REAL (DOUBLE PRECISION in the D version) array of length at least  $n$  in which the subroutine will set the first  $n$  elements to the differences between the initial approximations to  $y(x_i)$   $i=1,2,\dots,n$  and the final values, i.e. the elements of E will show the effect of the third and fourth difference corrections. The elements of E will be set to zero on return from a call with LIM=0.
- LIM is an INTEGER variable which the user must set to a limit on the number of difference corrections the subroutine is allowed to apply. If no corrections are to be applied set LIM to zero.
- EPS is a REAL (DOUBLE PRECISION in the D version) variable and is set by the user. It is used by the subroutine to decide when the difference corrections have become insignificant. If  $e_i^{(k)} = y_i^{(k)} - y_i^{(k-1)}$   $i=1,2,\dots,n$  are the current corrections at the  $k$ th iteration, the approximations to the solution obtained so far are accepted as the final values if  $|e_i^{(k)}| \leq \epsilon$   $i=1,2,\dots,n$  where  $\epsilon \geq 0$  is to be specified in EPS. **N.B.** EPS is not relevant when LIM=0.
- K is an INTEGER variable and is set by the subroutine to the number of corrections that had to be applied at each point to achieve the final values. If the limit on the number of corrections, LIM, has been reached and the corrections are still significant an error return is made with K set to -1. If corrections were not requested, i.e. LIM=0, K is returned set to zero.
- W is a REAL (DOUBLE PRECISION in the D version) array of length at least  $8n$  which is used by the routine as workspace.

### 3 GENERAL INFORMATION

**Use of COMMON:** none.

**Workspace:** see argument W.

**Other subroutines:** MA35A/AD and TA03A/AD are called.

**Input/Output:** If the boundary conditions are incorrectly specified with  $a_1=b_1=0$  or  $a_n=b_n=0$  an error is diagnosed and a message printed (also a RETURN is made in that event).

**System dependence:** none.

**Restrictions:**  $n \geq 3$ .

### 4 METHOD

The differential equation is replaced by a finite difference approximation. An initial approximation to  $y(x)$  is obtained by ignoring 3rd and higher differences. Successive approximations are then obtained by applying correction terms based upon the 3rd and 4th differences of the previous values of  $y_i$ .

Correction terms for the ends of the range are found by assuming that 5th differences are negligible. Taylor's series is used when a boundary condition involves the first derivative term.

The method is given in L. Fox, Proc. Roy. Soc. A **190**, 31-59, 1947.