1 SUMMARY

Given a **real symmetric matrix A**=$\{a_{ij}\}_{n\times n}$ and an estimate λ_e of one of its eigenvalues, finds to a given accuracy **the eigenvalue nearest to** λ_e and its corresponding **eigenvector.** The eigenvector is normalized to have unit length and the user may provide the first estimate.

The power method is applied to the matrix $(\mathbf{A}-\lambda_e\mathbf{I})^{-1}$ with Aitken extrapolation every 3rd iteration, see 'Modern Computing Methods', NPL.

ATTRIBUTES — Version: 1.0.0. **Types:** `EA02A; EA02AD.` **Calls:** `MB01.` **Original date:** July 1963. **Origin:** E.J.York*, Harwell.

2 HOW TO USE THE PACKAGE

2.1 Argument list and calling sequence

The single precision version:

```
CALL EA02A(A,N,IA,P,X,EPS)
```

The double precision version:

```
CALL EA02AD(A,N,IA,P,X,EPS)
```

`A` is a `REAL` (`DOUBLE PRECISION` in the D version) array of two dimensions in which the user must put the elements of the matrix **A**, i.e. `A(I,J)` = a_{ij}. This argument is not altered by the subroutine.

`N` is an `INTEGER` variable and is set by the user to *n* the order of the matrix. **Restriction:** $2 \le n \le 50$.

`IA` is an `INTEGER` variable and should be set by the user to the first dimension of the array `A`.

`P` is a `REAL` (`DOUBLE PRECISION` in the D version) variable and is set by the user to the value of the estimate λ_e of the required eigenvalue. On return the subroutine will have set `P` to the value of the eigenvalue nearest to the original estimate.

`X` is a `REAL` (`DOUBLE PRECISION` in the D version) array of length at least *n* in which the user may put a first estimate of the eigenvector. The default may be taken by setting the first *n* elements to zero, this will cause the subroutine to start with a vector having all its elements equal and set to $n^{-\frac{1}{2}}$. On return the array will contain the normalized eigenvector corresponding to the eigenvalue returned in `P`.

`EPS` is a `REAL` (`DOUBLE PRECISION` in the D version) variable and should be set by the user to the absolute accuracy required in the eigenvalue. See §4 for the error criterion used.

2.2 The Common area

The subroutine uses Common areas for workspace.

The single precision version:

```
COMMON/EA02C/ WORK(2750)
COMMON/EA02D/IWORK(50)
```

The double precision version:

```
COMMON/EA02CD/ WORK(2750)
COMMON/EA02DD/IWORK(50)
```

`WORK` is a `REAL` (`DOUBLE PRECISION` in the D version) array of length 2750 used as workspace. If *n*>50 then the array

must be of length at least $n*n+5*n$

IWORK is an INTEGER array of length 50 used as workspace. If $n>50$ then the array must be of length at least n.

3 GENERAL INFORMATION

Use of COMMON: labelled common areas EA02C/CD and EA02D/DD are used by the subroutine as workspace.

Workspace: only COMMON is used, see above.

Other subroutines: MB01 is called.

Input/Output: none.

System dependence: none.

Restrictions: $2 \leq n \leq 50$. The upper limit can be extended by re-compiling with larger dimensions for the workspace arrays in labelled common.

4 METHOD

Given an estimate λ_e to the required eigenvalue the matrix $\mathbf{M}=(\mathbf{A}-\lambda_e\mathbf{I})^{-1}$ is calculated, then an iterative scheme is used based upon

$$\mathbf{z}_n = \mathbf{M}\mathbf{x}_{n-1}$$

and

$$\mathbf{x}_n = s_n \mathbf{z}_n$$

where s_n is such that $\mathbf{x}_n$ will be normalized to have unit length. The Aitken extrapolation process is used at every third iteration to speed up convergence. When $|s_n - s_{n-1}| \leq \varepsilon$, where ε is the accuracy demanded by the user, the iterations are considered to be complete and the eigenvalue taken to be $\lambda_e + s_n$ and the eigenvector to be $\mathbf{x}_n$.