

PACKAGE SPECIFICATION

HSL ARCHIVE

**Warning:** Subroutine EA08 performs functions which are adequately treated by routines in other standard subroutine libraries (for example, LAPACK). The use of this routine is not recommended, and it may be removed from future releases of this library.

### 1 SUMMARY

Finds all the eigenvalues and eigenvectors of a real symmetric tri-diagonal matrix, i.e. finds eigenvalues  $\lambda_i$  and eigenvectors  $\mathbf{x}_i$ , i=1,2,...,m which are the nontrivial solutions of

$$Ax = \lambda x$$

for a matrix of the form

 $\mathbf{A} = \begin{bmatrix} a_1 & b_2 & & \\ b_2 & a_2 & b_3 & & \\ & b_3 & a_3 & b_4 & & \\ & * & * & * & \\ & & b_{m-1} & a_{m-1} & b_m \\ & & & & b_m & a_m \end{bmatrix}$ 

First EA09, which uses the QR algorithm, is used to find the eigenvalues, using these as shifts the QR algorithm is again applied but now using the plane rotations to generate the eigenvectors. Finally the eigenvalues are refined by taking Rayleigh quotients of the vectors.

ATTRIBUTES — Version: 1.0.0. Types: EA08C; EA08CD. Calls: EA09, FA05. Original date: February 1970. Origin: J.K.Reid, Harwell.

# **2 HOW TO USE THE PACKAGE**

#### 2.1 The argument list

The single precision version

CALL EA08C(A, B, VALUE, X, M, IX, W)

The double precision version

CALL EA08CD(A, B, VALUE, X, M, IX, W)

- A is a REAL (DOUBLE PRECISION in the D version) array of length at least m which must be set by the user to the diagonal elements  $a_i$ , i=1,2,...,m of the matrix.
- B is a REAL (DOUBLE PRECISION in the D version) array of length at least m which the user must set to the off-diagonal elements  $b_i$ , i=2,3,...,m of the matrix as defined in §1. Note that B(1) is not set.
- VALUE is a REAL (DOUBLE PRECISION in the D version) array of length at least *m* which will be set by the subroutine to the eigenvalues  $\lambda_i$ , *i*=1,2,...,*m*
- X is a REAL (DOUBLE PRECISION in the D version) array of dimension at least  $m \times m$  (first dimension IX) in which the subroutine returns the eigenvectors. The components of the eigenvector  $\mathbf{x}_i$  corresponding to the eigenvalue  $\lambda_i$  (in VALUE(I)) are placed in X(J,I) for J=1,...,M. The eigenvectors are normalized so that  $\mathbf{x}_i^T \mathbf{x}_i = 1$ , i=1,2,...,m.

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- M is an INTEGER variable which must be set by the user to *m* the order of the matrix.
- IX is an INTEGER variable which must be set by the user to the first dimension of the array X.
- W is a REAL (DOUBLE PRECISION in the D version) array of length at least 2m which is used by the subroutine as workspace.

## **3** GENERAL INFORMATION

Use of common: None.

Workspace: See argument W.

**Other routines called directly:** EA09C/CD.

Input/output: None.

# 4 METHOD

First EA09C/CD is called to find the eigenvalues and then the QR algorithm is repeated with known eigenvalues as shifts. During the second application of QR the plane rotations are multiplied into a matrix which eventually holds the eigenvectors. Finally the eigenvalues are refined by taking the Rayleigh quotients of the eigenvectors.