Warning: Subroutine EA11 performs functions which are adequately treated by routines in other standard subroutine libraries (for example, LAPACK). The use of this routine is not recommended, and it may be removed from future releases of this library.

1 SUMMARY

Calculates all the **eigenvalues** λ_i and **eigenvectors** $\mathbf{x}_i$ of the system $\mathbf{A}\mathbf{x}_i = \lambda_i \mathbf{B}\mathbf{x}_i$ where $\mathbf{A}=\{a_{ij}\}_{n\times n}$ is a real symmetric matrix and $\mathbf{B}=\{a_{ij}\}_{n\times n}$ is a real symmetric positive definite matrix.

The matrix $\mathbf{B}$ is factorized into $\mathbf{L}\mathbf{L}^T$ and the eigenvalue problem $\mathbf{L}^{-1}\mathbf{A}(\mathbf{L}^T)^{-1}\mathbf{L}^T\mathbf{x} = \mathbf{L}^T\mathbf{x}$ is solved using the EA06-EA09 subroutines.

ATTRIBUTES — **Version:** 1.0.0. **Types:** EA11A; EA11AD. **Calls:** EA06 and MA22. **Original date:** February 1972. **Origin:** S.Marlow, Harwell.

2 HOW TO USE THE PACKAGE

2.1 Argument list and calling sequence

The single precision version:

```
CALL EA11A(A,B,C,ID,D,N,W)
```

The double precision version:

```
CALL EA11AD(A,B,C,ID,D,N,W)
```

A is a two-dimensional REAL (DOUBLE PRECISION in the D version) array in which the user must set the coefficients a_{ij} of the matrix **A**. This array is overwritten by the subroutine.

B is a two-dimensional REAL (DOUBLE PRECISION in the D version) array in which the user must set the coefficients b_{ij} of the matrix **B**. This array is overwritten by the subroutine.

C is a two-dimensional REAL (DOUBLE PRECISION in the D version) array which will contain the eigenvectors on a successful exit from the subroutine. The components of the eigenvector $\mathbf{x}_i$ corresponding to the eigenvalue λ_i will be found in C(J,I), J $=1,2,.,n$ where I $=i$. The eigenvectors are normalized so as to have an Euclidean norm of one with the largest element positive.

ID is an INTEGER variable and must be set by the user to the first dimension of the arrays A, B and C.

D is a REAL (DOUBLE PRECISION in the D version) array which will contain the eigenvalues on a successful exit from the subroutine. The i^{th} eigenvalue λ_i will be found in D(I).

N is an INTEGER variable and must be set by the user to n the order of the matrices **A** and **B**.

W is a REAL (DOUBLE PRECISION in the D version) array of length at least $5n$ used as workspace by the subroutine. On exit, W(1) will contain an error flag and have one of the following values
- 0. successful entry
- –1. matrix **B** found to be not positive definite
- –2. value of N less than one.

3 GENERAL INFORMATION

Use of COMMON: none.

Workspace: provided by the user in `W`, $5n$ words.

Other subroutines: the subroutine calls `EA06C/CD`, `MA22A/AD`.

Input/Output: A diagnostic is printed if the value of `N` is less than 1 or the matrix **B** is not positive definite.

System dependence: none.

Restrictions:

$n \geq 1$,

B must be positive definite

4 METHOD

The equation in the summary can be represented as $\mathbf{AV} = \mathbf{BVE}$ where $\mathbf{V}$ is the matrix of eigenvectors and $\mathbf{E}$ is a diagonal matrix of the eigenvalues.

Since $\mathbf{B}$ is positive definite, it can be factorised as $\mathbf{B} = \mathbf{LDL}^T$ where $\mathbf{L}$ is a unit lower triangle matrix and $\mathbf{D}$ is a diagonal matrix of positive elements.

$\mathbf{AV} = \mathbf{BVE}$ can be transformed into $\mathbf{HG} = \mathbf{GE}$ where $\mathbf{H} = \mathbf{FAF}^T$ is a symmetric matrix and $\mathbf{G} = (\mathbf{F}^T)^{-1}\mathbf{V}$ is its corresponding matrix of eigenvectors, and where $\mathbf{F} = \mathbf{D}^{-\frac{1}{2}}\mathbf{L}^{-1}$. The eigenvectors and eigenvalues of the system $\mathbf{HG} = \mathbf{GE}$ are found. The eigenvectors of the original system are then found by the transformed $\mathbf{V} = \mathbf{F}^T\mathbf{G}$.