1 SUMMARY

To **evaluate the function** $e^{-z^2}\operatorname{erfc}(-iz)$ **for complex** z, viz.

$$W(z) = e^{-z^2}\left(1 + \frac{2i}{\sqrt{\pi}}\int_0^z e^{t^2}\,dt\right)$$

$W(z)$ is evaluated for $z=x+iy$, $x, y > 0$ using the approximations

(a) if $|z| \le 1$ integrate first few terms of expansion of e^{t^2}.

(b) if $1 < |z| \le 4$ and $y < 1.4$ integrate the Taylor series expansion of e^{t^2} about points in the region at which the value of $W(z)$ is known.

(c) if $1 < |z|$ and $y > 1.4$ numerically integrate

$$W(z) = \frac{i}{\pi}\int_{-\infty}^{\infty} \frac{e^{-t^2}}{z-t}\,dt$$

using the midpoint rule or Gauss-Hermite.

For outside the region $x, y > 0$ the relations

$W(-z) = 2e^{-z^2} - W(z)$ and $W(\bar{z}) = \overline{W}(z)$ are used.

ATTRIBUTES — **Version:** 1.0.0. **Types:** `FC01A`; `FC01AD`. **Original date:** February 1964. **Origin:** A.Bailey*, Harwell.

2 HOW TO USE THE PACKAGE

The single precision version

```
CALL FC01A(X,Y,U,V)
```

The double precision version

```
CALL FC01AD(X,Y,U,V)
```

`X` is a `REAL` (`DOUBLE PRECISION` in the D version) variable which must be set by the user to x the real part of the argument $z=x+iy$. **Restriction:** $x > 0$.

`Y` is a `REAL` (`DOUBLE PRECISION` in the D version) variable which must be set by the user to y the imaginary part of the argument $z=x+iy$. **Restriction:** $y > 0$.

`U` is a `REAL` (`DOUBLE PRECISION` in the D version) variable which is set by the subroutine to the real part of the computed value of the function $W(z)$.

`V` is a `REAL` (`DOUBLE PRECISION` in the D version) variable which is set by the subroutine to the imaginary part of the computed value of the function $W(z)$.

3 GENERAL INFORMATION

Use of common: none.

Workspace: none.

Other subroutines: none.

Input/Output: none.

Restrictions:

$x > 0$,

$y > 0$.

4 METHOD

$W(z)$ is evaluated for z in the first quadrant, other values being calculated from

$$W(-z) = 2e^{z^2} - W(z)$$

and

$$W(\bar{z}) = \bar{W}(-z)$$

Within the first quadrant the approximations are computed as follows

(a) If $|z| < 1$, then $W(z)$ is calculated by integrating the first few terms of the expansion of e^{t^2}.

(b) If $1 < |z| < 4$ and $y < 1.4$ integration of a Taylor series expansion of e^{t^2} about one of a number of fixed points in the region, at which the value of $W(z)$ is known.

(c) If $1 < |z| < 4$ and $y > 1.4$ the integral

$$W(z) = \frac{i}{\pi} \int_{-\infty}^{\infty} \frac{e^{-t^2}}{z-t} \, dt$$

is calculated using the midpoint rule.

(d) If $|z| > 4$ the above integral representation is calculated using a Gauss-Hermite quadrature formula.

At the boundaries of the various regions, the function is calculated by two methods and linear interpolation is used to give a continuous error curve.