1 SUMMARY

Computes the real and imaginary parts of the **Plasma Dispersion Function**

$$Z(z) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-t^2}}{t-z}\, dt$$

where $z = x+iy$, for the case $y > 0$, and the analytic continuation of this for $y < 0$ as defined by Fried and Conte, 'The Plasma Dispersion Function' , Academic Press, 1961. The derivative $Z'(z) = -2\times(1+zZ(z))$ is also computed.

If $y \geq 2.75$ or if $y \geq 2$ and $x \geq 4$ an asymptotic continued fraction due to Fried and Conte is used, otherwise if $x \geq 6.25$ a rational approximation from Abramowitz and Stegun is used, otherwise a Taylor series is used.

ATTRIBUTES — **Version:** 1.0.0. **Types:** FC12A; FC12AD. **Original date:** March 1973. **Origin:** R.Fletcher*, Harwell.

2 HOW TO USE THE PACKAGE

The single precision version

```
CALL FC12A(X,Y,ZR,ZI,ZPR,ZPI)
```

The double precision version

```
CALL FC12AD(X,Y,ZR,ZI,ZPR,ZPI)
```

X is a REAL (DOUBLE PRECISION in the D version) variable which must be set by the user to the value of x the real part of the argument z=x+iy.

Y is a REAL (DOUBLE PRECISION in the D version) variable which must be set by the user to the value of y the imaginary part of the argument z=x+iy.

ZR is a REAL (DOUBLE PRECISION in the D version) variable which is set by the subroutine to the real part of the computed value of the function $Z(z)$.

ZI is a REAL (DOUBLE PRECISION in the D version) variable which is set by the subroutine to the imaginary part of the computed value of the function $Z(z)$.

ZPR is a REAL (DOUBLE PRECISION in the D version) variable which is set by the subroutine to the real part of the computed value of the first derivative $Z'(z)$.

ZPI is a REAL (DOUBLE PRECISION in the D version) variable which is set by the subroutine to the imaginary part of the computed value of the first derivative $Z'(z)$.

3 GENERAL INFORMATION

Use of common: none.

Workspace: none.

Other subroutines: none.

Input/Output: none.

Restrictions: none.

Accuracies: approx. 10^{-6} absolute.

4 METHOD

For $x, y \geq 0$ one of the following three methods is used

(i) If $y \geq 2.75$, or if $y \geq 2$ and $x \geq 4$: the subroutine uses the asymptotic continued fraction given by B.D.Fried and S.D.Conte, ''The Plasma Dispersion Function'', Academic Press, 1961. {Note that some of the signs in the tables for $y < 0$ are wrong in the report from which this book was derived.}

(ii) Otherwise if $x \geq 6.25$: the subroutine uses the rational function

$$Z(z) = -z \left(\frac{0.9082482}{z^2 - 0.2752551} + \frac{0.09175171}{z^2 - 2.724745} \right)$$

based on one given in M.Abramowitz and I.A.Stegun, ''Handbook of Mathematical Functions'', Dover, 1965.

(iii) Otherwise: the subroutine uses a Taylor series expansion

$$Z(z) = \sum_{i=0} T(z)^{(i)}$$

about the nearest point z_0 on the mesh $x_0 = 0.0(0.5)6.0$, $y_0 = 0.0(0.5)2.5$ using the recurrences

$$T^{(0)} = Z(z_0), \qquad T^{(1)} = (z - z_0)\, Z'(z_0)$$

$$T^{(n+2)} = -(z - z_0) \left\{ \frac{z_0 T^{(n+1)} + (z - z_0) T^{(n)}}{\tfrac{1}{2}n + 1} \right\} \qquad n \geq 0$$

For $y < 0$ with $x > 0$ the relationship

$$Z(x - i|y|) = 2\sqrt{\pi}\, e^{y^2 - x^2} \{-\sin(2x|y|) + ...$$

$$...+ i \cos(2x|y|)\} + \mathrm{conj}\{Z(x + i|y|)\}$$

is used, and for $x < 0$ the relationship

$$Z(-|x| + iy) = -\mathrm{conj}\{Z(x + i|y|)\}$$

is used. The boundaries are chosen so that the results agree with the tables in Fried and Conte on a mesh $x = \pm 0.0(0.2)7.0$, $y = 0.0(0.2)4.0$ to an absolute accuracy in both real$\{Z(z)\}$ and imag$\{Z(z)\}$ of 10^{-6} or better. There is good reason to think that a similar accuracy holds for larger x and y. For $y < 0$ the error is dominated by the first term of the formula (4.1) and is better than about $2 \times 10^{-6} \times 2\sqrt{\pi} \exp(y^2 - x^2)$.