1 SUMMARY

To evaluate the **complement** of the cumulative distribution function of the **variance ratio distribution** with (n,m) degrees of freedom.

$$P(n,m,F) = \frac{n^{\frac{n}{2}} m^{\frac{m}{2}}}{B(\frac{n}{2},\frac{m}{2})} \int_F^{\infty} \frac{f^{\frac{n-2}{2}}}{(nf+m)^{\frac{n+m}{2}}} df \quad 0 \le F \le \infty$$

where n and m are positive integers.

Series expansions in $\sin\alpha$ and $\cos\alpha$ are used for the integral, where $\alpha = \tan^{-1}\sqrt{\frac{nF}{m}}$.

ATTRIBUTES — Version: 1.0.0. **Types:** `SA03A`; `SA03AD`. **Original date:** December 1970. **Origin:** D.G.Papworth, MRC, Harwell.

2 HOW TO USE THE PACKAGE

2.1 Argument list

The single precision version

```
CALL SA03A(F,N,M,P)
```

The double precision version

```
CALL SA03AD(F,N,M,P)
```

`F` is a `REAL` (`DOUBLE PRECISION` in the D version) variable which must be set by the user to the F value

`N` is an `INTEGER` which must be set by the user to n the first number of degrees of freedom. **Restriction:** $n > 0$.

`M` is an `INTEGER` which must be set by the user to m the second number of degrees of freedom. **Restriction:** $m > 0$.

`P` is a `REAL` (`DOUBLE PRECISION` in the D version) variable which is set by the subroutine to the value of $P(n,m,F)$.

3 GENERAL INFORMATION

Use of common: none.

Workspace: none.

Other routines called directly: none.

Input/output: none.

Restrictions: $n > 0$, $m > 0$.

4 METHOD

A series expansion for the integral is used. Let $\alpha = \tan^{-1}\sqrt{\frac{nF}{m}}$, then if n is even

$$P(n,m,F) = \cos^m\alpha\left\{1 + \frac{m}{2}\sin^2\alpha + \frac{m(m+2)}{2.4}\sin^4\alpha + \ldots + \frac{m(m+2)\ldots(m+n-4)}{2.4\ldots(n-2)}\sin^{n-2}\alpha\right\}$$

If m is even,

$$P(n,m,F) = 1 - \sin^n\alpha\left\{1 + \frac{n}{2}\cos^2\alpha + \frac{n(n+2)}{2.4}\cos^4\alpha + \ldots + \frac{n(n+2)\ldots(n+m-4)}{2.4\ldots(m-2)}\cos^{m-2}\alpha\right\}$$

If n and m are both odd

$$P(n,m,F) = \frac{2}{\pi}\frac{2.4\ldots(m-1)}{1.3\ldots(m-2)}\cos^m\alpha\sin\alpha\left\{1 + \frac{m+1}{3}\sin^2\alpha + \frac{(m+1)(m+3)}{3.5}\sin^4\alpha + \ldots\right.$$

$$\left.\ldots + \frac{(m+1)(m+3)\ldots(m+n-4)}{3.5\ldots(n-2)}\sin^{n-3}\alpha\right\}$$

$$- \frac{2\sin\alpha\cos\alpha}{\pi}\left\{1 + \frac{2}{3}\cos^2\alpha + \frac{2.4}{3.5}\cos^4\alpha + \ldots + \frac{2.4\ldots(m-3)}{3.5\ldots(m-2)}\cos^{m-3}\alpha\right\} + 1 - \frac{2\alpha}{\pi}$$

where, if $n=1$, the first series is to be taken as zero, and if $m=1$, the second series is to be taken as zero and the factor

$$\frac{2.4\ldots(m-1)}{3.5\ldots(m-2)}$$

is to be taken as unity.